10

Psychrometrics

10.1. Concept of psychrometry and psychrometrics. 10.2. Definitions. 10.3. Psychrometric relations. 10.4. Psychrometers. 10.5. Psychrometric charts. 10.6. Psychrometric processes : Mixing of air streams—Sensible heating—Sensible cooling—Cooling and dehumidification—Cooling and humidification—Heating and dehumidification—Heating and humidification—Highlights—Objective Type Questions—Theoretical Questions—Unsolved Examples.

10.1. CONCEPT OF PSYCHROMETRY AND PSYCHROMETRICS

Air comprises of fixed gases principally, nitrogen and oxygen with an admixture of water vapour in varying amounts. In atmospheric air water is always present and its relative weight averages less than 1% of the weight of atmospheric air in temperate climates and less than 3% by weight under the most extreme natural climatic conditions, it is nevertheless one of most important factors in human comfort and has significant effects on many materials. Its effect on human activities is in fact altogether disproportionate to its relative weights. *The art of measuring* the *moisture content of air is termed* "**psychrometry**". *The science which investigates the thermal properties of moist air, considers the measurement and control of the moisture content of air, and studies the effect of atmospheric moisture on material and human comfort may properly be termed* "**psychrometrics**".

10.2. DEFINITIONS

Some of the more important definitions are given below :

1. **Dry air.** The international joint committee on Psychrometric Data has adopted the following exact composition of air expressed in mole fractions (Volumetric) Oxygen 0.2095, Nitrogen 0.7809, Argon 0.0093, Carbon dioxide 0.0003. Traces of rare gases are neglected. Molecular weight of air for all air conditioning calculations will be taken as 28.97. Hence the gas constant,

$$R_{air} = \frac{8.3143}{28.97} = 0.287 \text{ kJ/kg K}$$

Dry air is never found in practice. Air always contains some moisture. Hence the common designation "air" *usually means moist* air. The term *'dry air'* is used to indicate the *water free contents* of air having any degree of moisture.

2. **Saturated air.** Moist air is said to be saturated when its condition is such that it can co-exist in natural equilibrium with an associated condensed moisture phase presenting a flat surface to it. For *a given temperature, a given quantity of air can be saturated with a fixed quantity of moisture.* At higher temperatures, it requires a larger quantity of moisture to saturate it. At saturation, vapour pressure of moisture in air corresponds to the saturation pressure given in steam tables corresponding to the given temperature of air.

3. **Dry-bulb temperature (DBT).** It *is the temperature of air as registered by an ordinary thermometer* (t_{db}).

4. **Wet-bulb temperature (WBT).** *It is the temperature registered by a thermometer when the bulb is covered by a wetted wick and is exposed to a current of rapidly moving air* (t_{wb}).

5. **Adiabatic saturation temperature.** *It is the temperature at which the water or ice can saturate air by evaporating adiabatically into it.* It is numerically equivalent to the measured wet bulb temperature (as corrected, if necessary for radiation and conduction) (t_{wb}).

6. **Wet bulb depression.** It *is the difference between dry-bulb and wet bulb temperatures* ($t_{db} - t_{wb}$).

7. **Dew point temperature (DPT).** *It is the temperature to which air must be cooled at constant pressure in order to cause condensation of any of its water vapour.* It is equal to steam table saturation temperature corresponding to the actual partial pressure of water vapour in the air (t_{dp}).

8. **Dew point depression.** *It* is *the difference between the dry bulb and dew point temperatures* ($t_{db} - t_{dp}$).

9. **Specific humidity (Humidity ratio).** *It is the ratio of the mass of water vapour per unit mass of dry air in the mixture of vapour and air,* it is generally expressed as grams of water per kg of dry air. For a given barometric pressure it is a function of dew point temperature alone.

10. **Relative humidity (RH), (ϕ).** *It is the ratio of the partial pressure of water vapour in the mixture to the saturated partial pressure at the dry bulb temperature, expressed as percentage.*

11. **Sensible heat.** *It is the heat that changes the temperature of a substance when added to or abstracted from it.*

12. **Latent heat.** *It is the heat that does not affect the temperature but changes the state of substance when added to or abstracted from it.*

13. **Enthalpy.** *It is the combination energy which represents the sum of internal and flow energy in a steady flow process.* It is determined from an arbitrary datum point for the air mixture and is expressed as kJ per kg of dry air (h).

Note. When air is saturated DBT, WBT, DPT are *equal.*

10.3. PSYCHROMETRIC RELATIONS

Pressure

Dalton's law of partial pressure is employed to determine the pressure of a mixture of gases. This law *states that the total pressure of a mixture of gases is equal to the sum of partial pressures which the component gases would exert if each existed alone in the mixture volume at the mixture temperature.*

Precise measurements made during the last few years indicate that this law as well as Boyle's and Charle's laws are only approximately correct. Modern tables of atmospheric air properties are based on the correct versions.

For calculating partial pressure of water vapour in the air many equations have been proposed, probably Dr. Carrier's equation is most widely used.

$$p_v = (p_{vs})_{wb} - \frac{[pt - (p_{vs})_{wb}](t_{db} - t_{wb})}{1527.4 - 1.3\, t_{wb}} \quad \text{...(10.1)}$$

where p_v = Partial pressure of water vapour,

p_{vs} = Partial pressure of water vapour when air is fully saturated,

p_t = Total pressure of moist air,

t_{db} = Dry bulb temperature (°C), and

t_{wb} = Wet bulb temperature (°C).

Specific humidity W :

Specific humidity $= \dfrac{\text{Mass of water vapour}}{\text{Mass of dry air}}$

or $$W = \frac{m_v}{m_a}$$

Also, $$m_a = \frac{p_a V}{R_a T} \qquad \text{...(10.2)}$$

and $$m_v = \frac{p_v \times V}{R_v \times T} \qquad \text{...(10.3)}$$

where p_a = Partial pressure of dry air,

p_v = Partial pressure of water vapour,

V = Volume of mixture,

R_a = Characteristic gas constant for dry air, and

R_v = Characteristic gas constant for water vapour.

From equations (10.2) and (10.3)

$$W = \frac{p_v \times V}{R_v \times T} \times \frac{R_a T}{p_a V} = \frac{R_a}{R_v} \times \frac{p_v}{p_a}$$

But $$R_a = \frac{R_0}{M_a} \qquad \left(= \frac{8.3143}{28.97} = 0.287 \text{ kJ/kg K in SI units} \right)$$

$$R_v = \frac{R_0}{M_v} \qquad \left(= \frac{8.3143}{18} = 0.462 \text{ kJ/kg K in SI units} \right)$$

where R_0 = Universal gas constant,

M_a = Molecular weight of air, and

M_v = Molecular weight of water vapour.

$$\therefore \qquad W = \frac{0.287}{0.462} . \frac{p_v}{p_a} = 0.622 \; \frac{p_v}{p_t - p_v}$$

i.e., $$W = 0.622 \; \frac{p_v}{p_t - p_v} \qquad \text{...(10.4)}$$

The masses of air and water vapour in terms of specific volumes are given by expression as

$$m_a = \frac{V}{v_a} \quad \text{and} \quad m_v = \frac{V}{v_v}$$

where v_a = Specific volume of dry air, and

v_v = Specific volume of water vapour.

$$W = \frac{v_a}{v_v} \qquad \text{...(10.5)}$$

Degree of saturation (μ) :

$$\text{Degree of saturation} = \frac{\begin{array}{c}\text{Mass of water vapour associated}\\ \text{with unit mass of dry air}\end{array}}{\begin{array}{c}\text{Mass of water vapour associated with}\\ \text{saturated unit mass of dry saturated air}\end{array}}$$

i.e.,
$$\mu = \frac{W}{W_s} \qquad \text{...(10.6)}$$

where, W_s = Specific humidity of air when air is fully saturated

$$\therefore \qquad \mu = \frac{0.622\left(\dfrac{p_v}{p_t - p_v}\right)}{0.622\left(\dfrac{p_{vs}}{p_t - p_{vs}}\right)} = \frac{p_v(p_t - p_{vs})}{p_{vs}(p_t - p_v)}$$

$$= \frac{p_v}{p_s}\left[\frac{\left(1 - \dfrac{p_{vs}}{p_t}\right)}{\left(1 - \dfrac{p_v}{p_t}\right)}\right] \qquad \text{...(10.7)}$$

where p_{vs} = Partial pressure of water vapour when air is fully saturated (p_{vs} can be calculated from steam tables corresponding to the dry bulb temperature of the air).

Relative humidity (RH), ϕ :

$$\text{Relative humidity, } \phi = \frac{\text{Mass of water vapour in a given volume}}{\text{Mass of water vapour in the same volume if saturated at the same temp.}}$$

$$= \frac{m}{m_{vs}} = \frac{\dfrac{p_v T}{R_v T}}{\dfrac{p_{vs} T}{R_v T}} = \frac{p_v}{p_{vs}} \qquad \text{...(10.8)}$$

Inserting the value of equation (10.8) into equation (10.7), we get

$$\mu = \phi\left[\frac{1 - \dfrac{p_{vs}}{p_v}}{1 - \dfrac{p_v}{p_t}}\right] = \phi\left[\frac{1 - \dfrac{p_{vs}}{p_t}}{1 - \dfrac{\phi p_{vs}}{p_t}}\right] = \phi\left(\frac{p_t - p_{vs}}{p_t - \phi\, p_{vs}}\right)$$

$$\therefore \qquad \phi(p_t - p_{vs}) = \mu(p_t - \phi\, p_{vs})$$

or
$$\phi(p_t - p_{vs} + \mu p_{vs}) = \mu p_t$$

$$\therefore \qquad \phi = \frac{\mu p_t}{p_t - p_{vs} + \mu p_{vs}} = \frac{\mu}{1 - (1 - \mu)\dfrac{p_{vs}}{p_t}} \qquad \text{...(10.9)}$$

Since $p_{vs} << p_t$

$\therefore$ $\phi \simeq \mu$

Inserting the value of the p_v from equation (10.4) into equation (10.8), we get

$$\phi = \frac{p_a W}{0.622} \times \frac{1}{p_{vs}} = 1.6\ W\,\frac{p_a}{p_{vs}} \qquad \text{...(10.10)}$$

Note 1. Relative humidity as compared to specific humidity plays a vital role in comfort air-conditioning and industrial air-conditioning. *Relative humidity signifies the absorption capacity of air.* If initial relative humidity of air is less it will absorb more moisture.

2. W, μ and ϕ cannot be conveniently measured as they require measurement of p_v and p_{vs}. The value of p_v can be obtained from the measurement of the wet bulb temperature and the value of p_{vs} can be calculated from steam tables corresponding to given air temperature.

Enthalpy of moist air

It is the sum of enthalpy of dry air and enthalpy of water vapour associated with dry air. It is expressed in kJ/kg of dry air

$$h = h_{\text{air}} + W \cdot h_{\text{vapour}}$$
$$= c_p t_{db} + W \cdot h_{\text{vapour}}$$

where h = Enthalpy of mixture/kg of dry air,

h_{air} = Enthalpy of 1 kg of dry air,

h_{vapour} = Enthalpy of 1 kg of vapour obtained from steam tables,

W = Specific humidity in kg/kg of dry air, and

c_p = Specific heat of dry air normally assumed as 1.005 kJ/kg K.

Also $h_{\text{vapour}} = h_g + c_{ps}(t_{db} - t_{dp})$

where h_g = Enthalpy of saturated steam at dew point temperature,

and c_{ps} = 1.88 kJ/kg K.

$$\therefore \quad h = c_p t_{db} + W[h_g + c_{ps}(t_{db} - t_{dp})] \qquad \text{...(10.11)}$$
$$= (c_p + c_{ps} W)\, t_{db} + W(h_g - c_{ps} t_{dp})$$
$$= c_{pm} t_{db} + W(h_g - c_{ps} t_{dp}) \qquad \text{...[10.11(a)]}$$

where $c_{pm} = (c_p + c_{ps} W)$ is the *specific heat of humid air or humid specific heat.*

The value of c_{pm} is taken as 1.021 kJ/kg dry air per K. It is the heat capacity of (1 + W) kg of moisture per kg of dry air.

$h_{\text{vapour}} \simeq h_g$ at dry bulb temperature. So,

$$h = c_p t_{db} + W h_g. \qquad \text{...(10.12)}$$

However, a better approximation is given by the following relationship :

$$h_{\text{vapour}} = 2500 + 1.88 t_{db} \text{ kJ/kg of water vapour} \qquad \text{...[10.12 (a)]}$$

where t_{db} is dry bulb temperature in °C, and the datum state is liquid water at 0°C.

$$\therefore \quad h = 1.005\, t_{db} + W(2500 + 1.88\, t_{db}) \text{ kJ/kg dry air.} \qquad \text{...[10.12 (b)]}$$

Adiabatic saturation process

In an insulated chamber when unsaturated air flows over a long sheet of water (Fig. 10.1), the water evaporates, and the specific humidity of the air increases. As the evaporation takes place both the air and water are cooled. *The process continues until the energy transferred from the air to the water is equal to the energy required to vapourise the water.* When this point is reached,

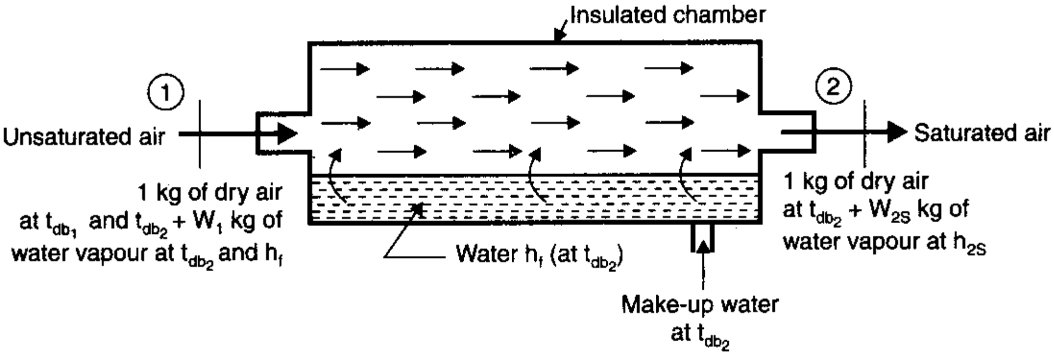


Fig. 10.1. Adiabatic saturation process.

thermal equilibrium exists with respect to water, air and water vapour, and consequently the air is saturated. The *equilibrium temperature is called the* ***adiabatic saturation temperature or the thermodynamic wet bulb temperature***. The make-up water is introduced at *this temperature* to make the water level constant. The '*adiabatic*' cooling process is shown in Fig. 10.2 for the vapour in the air-vapour mixture. Although the total pressure of the mixture is constant, the partial pressure of the vapour increases, and in the saturated state corresponds to the adiabatic saturation temperature. The vapour is initially at *DBT* t_{db_1} and is cooled adiabatically to *DBT* t_{db_2} which is equal to the adiabatic saturation t_{wb_2}. The adiabatic saturation temperature and wet bulb temperatures are taken to be equal for all practical purposes. The wet bulb temperature lies between the dry bulb temperature and dew point temperature.

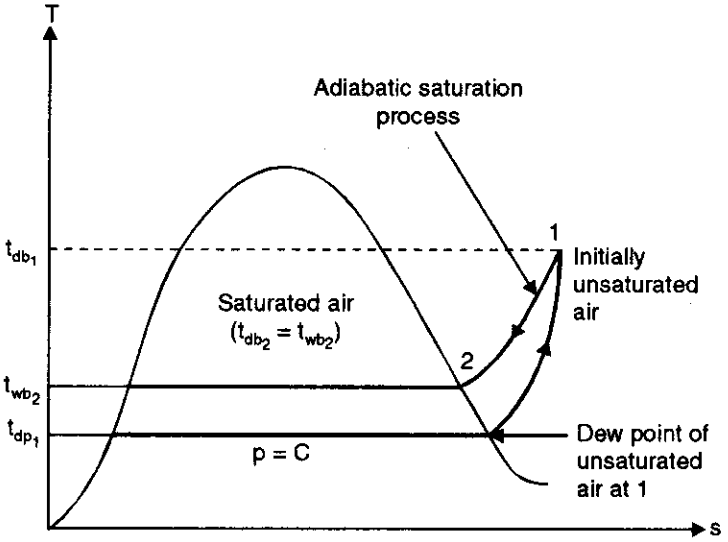


Fig. 10.2. Adiabatic cooling process.

Let us now apply the first law to the entire process. Considering the process to be steady state steady flow, neglecting changes in kinetic and potential energies, we have

$$h_1 + (W_{2s} - W_1)h_{f_2} = h_{2s} \qquad ...(10.13)$$

The quantities W_{2s}, h_{2s} and h_{f_2} are the functions of temperature t_{db_2}. The term $(W_{2s} - W_1)h_{f_2}$ is quite small and if this term is neglected, it can be seen that the enthalpy remains constant in adiabatic saturation.

Equation (10.13) may be rewritten as

$$h_1 - W_1h_{f_2} = h_{2s} - W_{2s}h_{f_2}$$

The inlet term can be generalized and the expression can be written as follows :

$$\Sigma = h_{2s} - W_{2s}h_{f_2} = h_1 - W_1h_{f_2} = h_x - W_xh_{f_2} \qquad ...(10.14)$$

This means that sigma function (Σ) as defined by the equation, is constant for any wet bulb temperature.

Also $$h_1 = h_{2s} - (W_{2s} - W_1)\, h_{f_2} \qquad ...(10.15)$$

Equation (10.15) indicates that the enthalpy of an air-water vapour mixture is equal to the enthalpy of saturated air at the same wet bulb temperature, less small correction term $(W_{2s} - W_1) h_{f_2}$. This correction term is called ***enthalpy deviation.***

$$h_1 = h_{\text{air (1)}} + W_1 \, h_{\text{vapour (1)}} \qquad \text{...(10.16)}$$

$$h_{2s} = h_{\text{air (2)}} + W_{2s} \, h_{\text{vapour (2)}} \qquad \text{...(10.17)}$$

or

$$h_{\text{air (1)}} + W_1 h_{g_1} + (W_{2s} - W_1) h_{f_2} = h_{\text{air (2)}} + W_{2s} h_{g_2}$$

$$c_p t_{db_1} + W_1 h_{g_1} + (W_{2s} - W_1) h_{f_2} = c_p t_{db_2} + W_{2s} h_{g_2} \qquad \text{...(10.18)}$$

After arranging, we get

$$W_1 = \frac{c_p (t_{db_2} - t_{db_1}) + W_{2s}(h_{g_2} - h_{f_2})}{h_{g_1} - h_{f_2}}$$

or

$$W_1 = \frac{c_p (t_{db_2} - t_{db_1}) + W_{2s} h_{g_2}}{h_{g_1} - h_{f_2}} \qquad \text{...[10.18(a)]}$$

Note. The wet bulb temperature is not a property of moist air as it is influenced by heat and mass transfer rates. Thus in psychrometric equations and psychrometric charts where the wet bulb temperature appears, *it is always the thermodynamic wet bulb temperature that is considered.*

10.4. PSYCHROMETERS

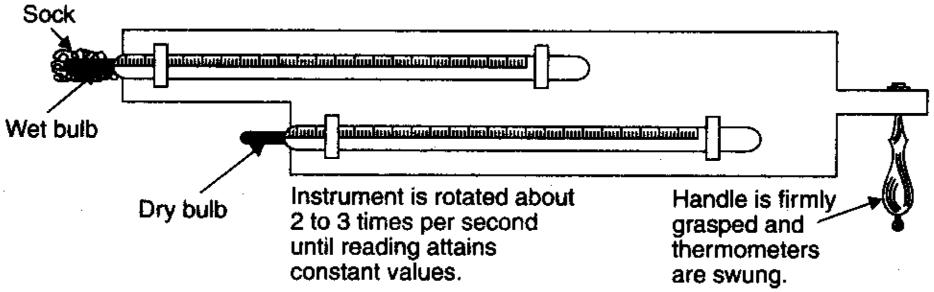
A psychrometer is a device which is used for measuring dry bulb and wet bulb temperatures simultaneously.

The psychrometers may be classified as follows :

1. Laboratory psychrometer
2. Sling psychrometer
3. Aspirating psychrometer
4. Continuous recording psychrometer.

The description of a sling psychrometer is given below :

Refer Fig. 10.3. The sling psychrometer consists of two thermometers mounted on a base plate. The one with the 'sock' is wet-bulb thermometer ; the other is dry-bulb. The wet bulb exists below the dry-bulb. This is done purposely so that 'sock' can be dipped in water without wetting the dry-bulb. The handle of the frame helps for rotating the psychrometer to produce necessary air motion. As the psychrometer is rotated it provides necessary air velocity over the thermometer. Fast movement of air past the 'sock' is necessary to bring the air at temperature t_{db} always in immediate contact with the wet 'sock'. The temperature spread between dry bulb and wet bulb readings depends upon the *amount of moisture in the air. Dry air*, or air that has low moisture



Instrument is rotated about 2 to 3 times per second until reading attains constant values.

Handle is firmly grasped and thermometers are swung.

Fig. 10.3. Sling psychrometer.

content has a low wet bulb temperature ; humid air that has a high moisture content, has a high wet-bulb temperature.

When dry and wet bulb temperatures are known the other psychrometric properties like relative humidity, dew point temperature, degree of saturation, humidity ratio, and volume of air-vapour mixture per kg of dry air are determined by calculations.

10.5. PSYCHROMETRIC CHARTS

The psychrometric charts are prepared to represent graphically all the necessary moist air properties used for air conditioning calculations. The values are based on actual measurements verified for thermodynamic consistency.

For psychrometric charts the most convenient co-ordinates are dry bulb temperature of air vapour mixture as the abcissa and moisture content (kg/kg of dry air) or water vapour pressure as the ordinate. Depending upon whether the humidity contents is abcissa or ordinate with temperature co-ordinate, the charts are generally classified as Mollier chart and Carrier chart. Carrier chart having t_{db} as the abcissa and W as the ordinate finds a wide application. The chart is constructed as under :

1. *The dry bulb temperature* (°C) of unit mass of dry air for different humidity contents or humidity ratios are indicated by *vertical lines drawn parallel to the ordinate.*

2. *The mass of water vapour* in kg (or grams) per kg of dry air is drawn parallel to the abcissa for different values of dry bulb temperature. It is the *major vertical scale of the chart.*

3. *Pressure of water vapour in mm of mercury* is shown in the scale at left and is the absolute pressure of steam.

4. *Dew point temperatures* are temperatures corresponding to the boiling points of water at low pressures of water vapour and are shown in the scale on the *upper curved line.* The dew points for different low pressures are read on *diagonal co-ordinates.*

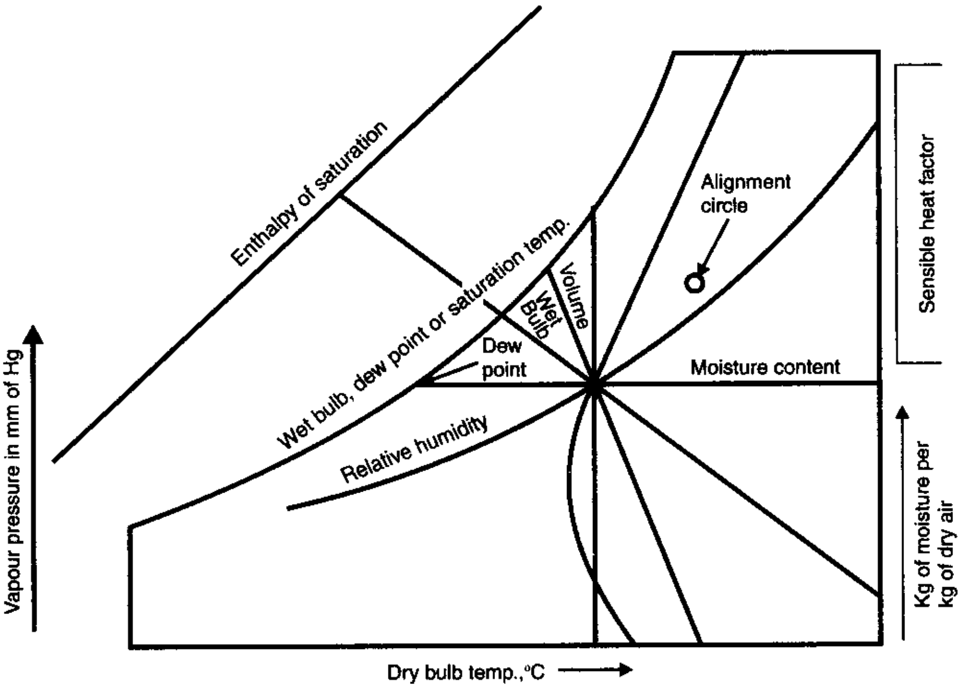


Fig. 10.4. Skeleton psychrometric chart.

5. Constant relative humidity lines in per cent are indicated by marking off vertical distances between the saturation line or the upper curved line and the base of the chart. The relative humidity curve depicts quantity (kg) of moisture actually present in the air as a percentage of the total amount possible at various dry bulb temperatures and masses of vapour.

6. *Enthalpy or total heat* at saturation temperature in kJ/kg of dry air is shown by a diagonal system of co-ordinates. The scale on the diagonal line is separate from the body of the chart and is indicated above the saturation line.

7. *Wet bulb temperatures* are shown on the diagonal co-ordinates coinciding with heat co-ordinates. The scale of wet bulb temperatures is shown on the saturation curve. The diagonals run downwards to the right at an angle of 30º to the horizontal.

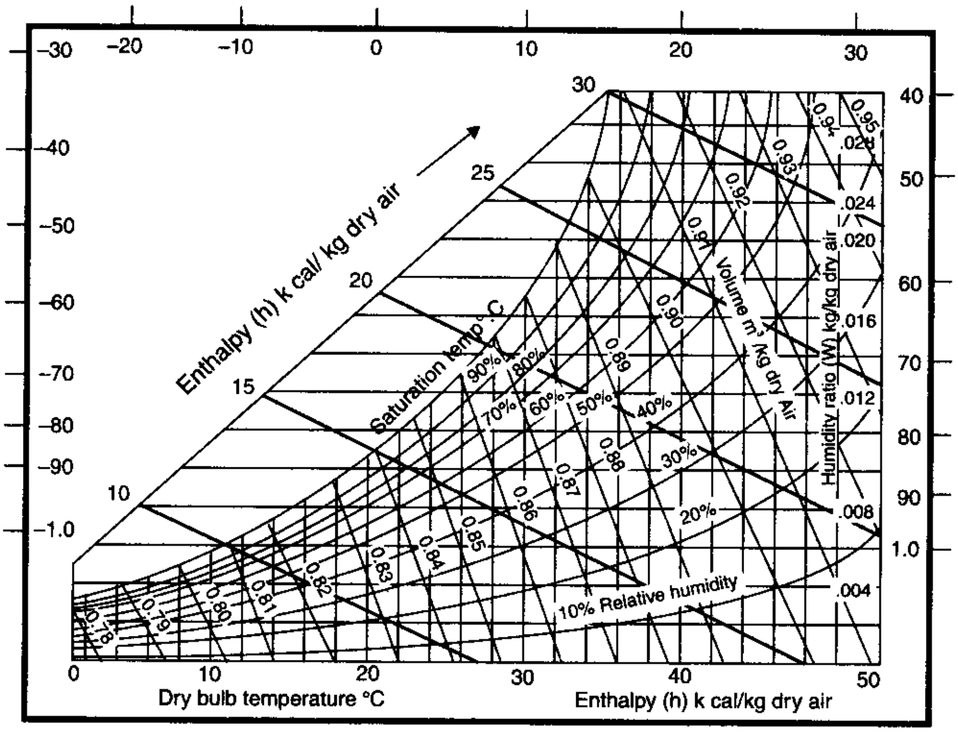


Fig. 10.5. Carrier chart.

8. The volume of air vapour mixture per kg of dry air (specific volume) is also indicated by a set of diagonal co-ordinates but at an angle of 60º with the horizontal.

The other properties of air vapour mixtures can be determined by using formulae (already discussed).

In relation to the psychrometric chart, these terms can quickly indicate many things about the condition of air, for example :

1. If dry bulb and wet bulb temperatures are known, the relative humidity can be read from the chart.

2. If the dry bulb and relative humidity are known, the wet bulb temperature can be determined.

3. If wet bulb temperature and relative humidity are known, the dry bulb temperature can be found.

4. If wet bulb and dry bulb temperatures are known, the dew point can be found.
5. If wet bulb and relative humidity are known, dew point can be read from the chart.
6. If dry-bulb and relative humidity are known, dew point can be found.
7. The quantity (kg) of moisture in air can be determined from any of the following combinations :

(*i*) Dry bulb temperature and relative humidity ;
(*ii*) Dry bulb temperature and dew point ;
(*iii*) Wet bulb temperature and relative humidity ;
(*iv*) Wet bulb temperature and dew point temperature ;
(*v*) Dry bulb temperature and wet bulb temperature ; and
(*vi*) Dew point temperature alone.

Figs. 10.4 and 10.5 show the skeleton psychrometric chart and lines on carrier chart respectively.

10.6. PSYCHROMETRIC PROCESSES

In order to condition air to the conditions of human comfort or of the optimum control of an industrial process required, certain processes are to be carried out on the outside air available. The processes affecting the *psychrometric properties of air are called* ***psychrometric processes.*** These processes involve mixing of air streams, heating, cooling, humidifying, dehumidifying, adiabatic saturation and mostly the combinations of these.

The important psychrometric processes are enumerated and explained in the following text :

1. Mixing of air streams
2. Sensible heating
3. Sensible cooling
4. Cooling and dehumidification
5. Cooling and humidification
6. Heating and dehumidification
7. Heating and humidification.

10.6.1. Mixing of Air Streams

Refer Figs. 10.6 and 10.7. Mixing of several air streams is the process which is very frequently used in air conditioning. This mixing normally takes place without the addition or rejection of either heat or moisture, *i.e., adiabatically and at constant total moisture content. Thus we can write the following equations :*

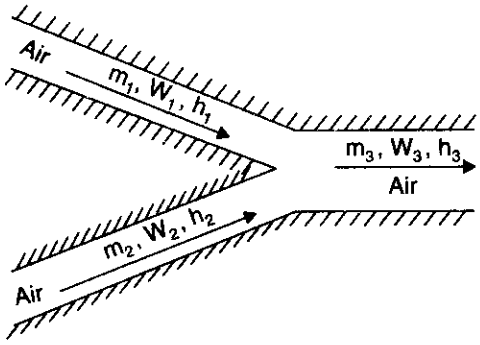


Fig. 10.6. Mixing of air streams.

$$m_1 + m_2 = m_3 \qquad ...(10.19)$$

$$m_1W_1 + m_2W_2 = m_3W_3 \qquad ...(10.20)$$

$$m_1h_1 + m_2h_2 = m_3h_3 \qquad ...(10.21)$$

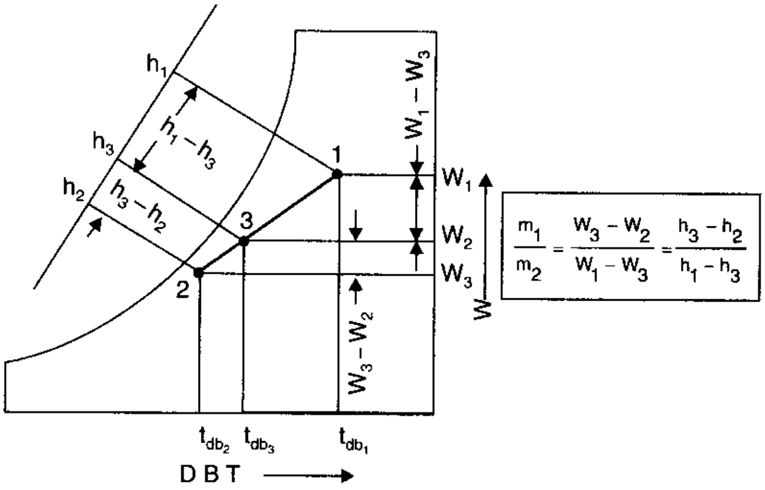


Fig. 10.7

Rearranging of last two equations gives the following :

$$m_1(W_1 - W_3) = m_2(W_3 - W_2)$$

$$m_1(h_1 - h_3) = m_2(h_3 - h_2)$$

or

$$\frac{m_1}{m_2} = \frac{W_3 - W_2}{W_1 - W_3} = \frac{h_3 - h_2}{h_1 - h_3}$$

where m = Mass of dry air, W = Specific humidity, h = Enthalpy at particular state points.

On the psychrometric chart, the specific humidity and enthalpy scales are linear, ignoring enthalpy deviations. Therefore, the final state 3 lies on a straight line connecting the initial states of the two streams before mixing, and the final state 3 divides this line into two parts that are in the same ratio as were the two masses of air before mixing.

If the air quantities are known in volume instead of mass units, it is generally sufficiently accurate to units of m^3 *or* $m^3/min.$ *in the mixing equations.* The inaccuracy introduced is due to the difference in specific volume at two initial states. This difference in densities is small for most of the comfort air conditioning problems.

10.6.2. Sensible Heating

When air passes over a dry surface which is at a temperature *greater* than its (air) dry bulb temperature, it undergoes *sensible heating.* Thus the heating can be achieved by passing the air over heating coil like electric resistance heating coils or steam coils. During such a process, the specific *humidity remains constant but the dry bulb temperature rises and approaches that of the surface.* The extent to which it approaches the mean effective surface temperature of the coil is conveniently expressed in terms of the equivalent **by-pass factor.**

The by-pass factor (BF) for the process is defined as the ratio of the difference between the mean surface temperature of the coil and leaving air temperature to the difference between the mean surface temperature and the entering air temperature. Thus on Fig. 10.8, air at temperature t_{db_1}, passes over a heating coil with an average surface temperature t_{db_3} and leaves at temperature t_{db_2} .

The by-pass factor is expressed as follows :

$$BF = \frac{t_{db_3} - t_{db_2}}{t_{db_3} - t_{db_1}} \qquad ...(10.22)$$

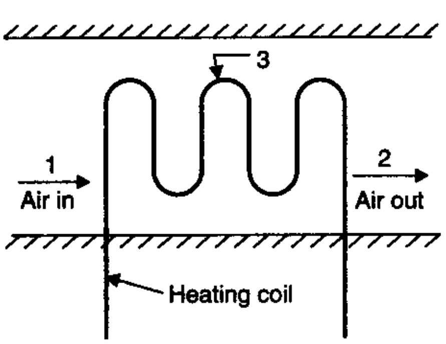


Fig. 10.8. Sensible heating.

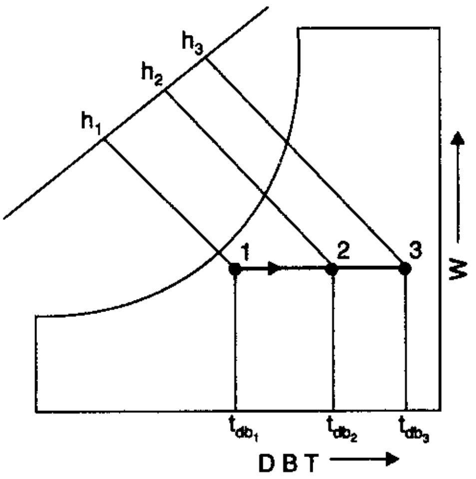


Fig. 10.9

Or in terms of lengths on the chart (Fig. 10.9) it is $\frac{\text{length } 2\text{-}3}{\text{length } 1\text{-}3}$. *The value of the by-pass factor is a function of coil design and velocity.* The heat added to the air can be obtained directly from the entering and leaving enthalpies ($h_2 - h_1$) or it can be obtained from the humid specific heat multiplied by the temperature difference ($t_{db_2} - t_{db_1}$).

In a complete air conditioning system the *preheating* and *reheating* of air are among the familar examples of sensible heating.

Note. 'By-pass factor' can be considered to represent the fraction of air which *does not* come into contact with coil surface.

10.6.3. Sensible Cooling

Refer Fig. 10.10. Air undergoes *sensible cooling* whenever it passes over a surface that is at a temperature less than the *dry bulb* temperature of the *air but greater than the dew point temperature.* Thus sensible cooling can be achieved by passing the air over cooling coil like *evaporating coil of the refrigeration cycle or secondary brine coil.* During the process, *the specific humidity remains constant* and *dry bulb temperature decreases,* approaching the mean effective surface temperature. On a psychrometric chart the process will appear as a horizonal line 1–2 (Fig. 10.11), where point 3 represents the effective surface temperature. For this process :

$$\text{By-pass factor BF} = \frac{t_{db_2} - t_{db_3}}{t_{db_1} - t_{db_3}} \qquad ...(10.23)$$

The heat removed from air can be obtained from the enthalpy difference ($h_1 - h_2$) or from humid specific heat multiplied by the temperature difference ($t_{db_1} - t_{db_2}$).

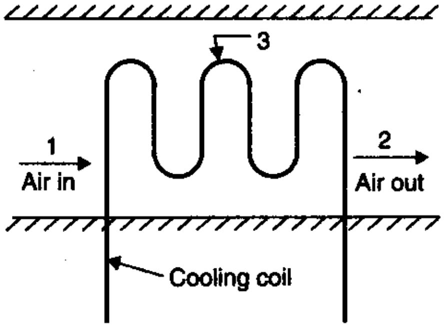


Fig. 10.10. Sensible cooling.

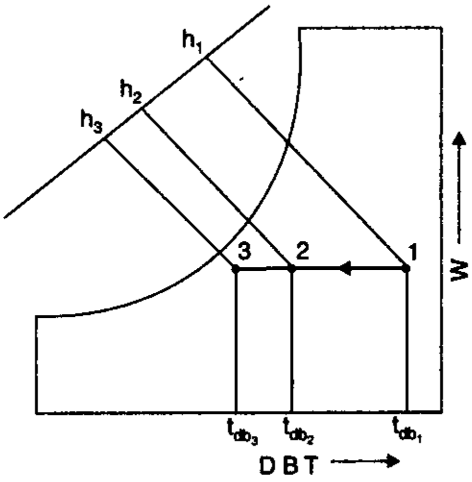


Fig. 10.11

10.6.4. Cooling and Dehumidification

Refer Fig. 10.12. Whenever air is made to pass over a surface or through a spray of water that is at a *temperature less than the dew point temperature of the air, condensation of some of the water vapour in air will occur simultaneously with the sensible cooling process.* Any air that

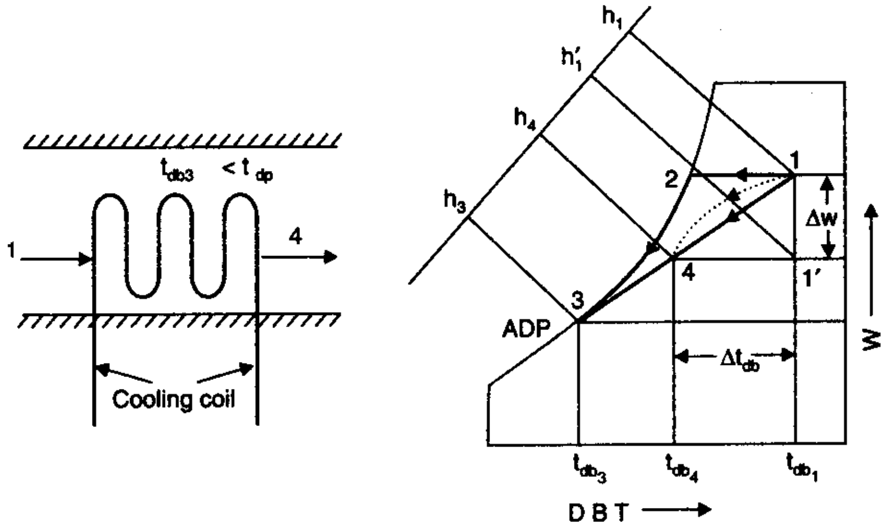


Fig. 10.12. Cooling and dehumidification.

comes into sufficient contact with the cooling surface will be reduced in temperature to the mean surface temperature along a path such as 1-2-3 in Fig. 10.12, with condensation and therefore dehumidification occurring between points 2 and 3. The air that does not contact the surface will be finally cooled by mixing with the portion that did, and the final state point will somewhere on the straight line connecting points 1 and 3. The actual path of air during the path will not be straight line shown but will be something similarly to the curved dashed line 1–4. It will result from a continuous mixing of air which is connecting a particular part of the coil and air which is by passing it. It is convenient, however to analyse the problem with the straight line shown, and to assume that the final air state results from the mixing of air that has completely by passed the coil with air that has been cooled to the mean effective surface temperature. If there is enough contact between air and surface for all the air to come to the mean surface temperature, the process is one of zero by pass. In any practical system, complete saturation is not obtained and final state will be a point such as 4 in Fig. 10.12 with an equivalent by pass factor equal to $\frac{\text{length } 3\text{-}4}{\text{length } 3\text{-}1}$. For processes involving condensation, the effective surface temperature, *e.g.* t_{db_3} in Fig. 10.12 is called **'apparatus dew point'** (ADP). The final state point of air passing through a cooling and dehumidifying apparatus is in effect a mixture condition that results from mixing the fraction of the air, which is equal to the equivalent by-pass factor (BF) and is at initial state point and the remaining fraction which is equal to one minus by pass factor (1–BF) and is saturated at the apparatus dew point (ADP).

Total heat removed from the air is given by

$$Q_t = h_1 - h_4 = (h_1 - h_1') + (h_1' - h_4)$$
$$= Q_L + Q_S$$

where,
W_L = Latent heat removed $(h_1 - h_1')$, and
Q_S = Sensible heat removed $(h_1' - h_4)$

The ratio $\frac{Q_S}{Q_L}$ *is called sensible heat factor* (SHF) Or *sensible heat ratio* (SHR)

$$\therefore \qquad \text{SHF} = \frac{Q_S}{Q_L + Q_S} \qquad \text{...(10.24)}$$

The ratio fixes the slope of the line 1—4 on the psychrometric chart. Sensible heat factor slope lines are given on the psychrometric chart. If the initial condition and SHF are known for the given process, then the process line can be drawn through the given initial condition at a slope given by SHF on the psychrometric chart.

The capacity of the cooling coil in *tonnes* of refrigeration is given by,

$$\text{Capacity in TR} = \frac{m_a(h_1 - h_4) \times 60}{14000}, \qquad \text{...(10.25)}$$

where m_a = mass of air, kg/min and h = enthalpy in kJ/kg of air.

10.6.5. Cooling and Humidification

If unsaturated air is passed through a spray of continuously recirculated water, the specific humidity will increase while the dry bulb temperature decreases. This is the process of ***adiabatic saturation*** or ***evaporative cooling.*** This process is one of constant adiabatic-saturation temperature and for all practical purposes, one of constant wet bulb temperature. The process is illustrated as path 1-2 on Fig. 10.13, with wet bulb temperature of air being that of point 3, which is also equilibrium temperature of the recirculated water if there is sufficient contact between air and spray, the air will leave at a condition very close to that of point 3. The concept of equivalent by pass can be applied to this process but another term is more used to describe the performance of a humidifying apparatus. It is the ***'saturating'*** or ***'humidifying efficiency'*** which is *defined as the*

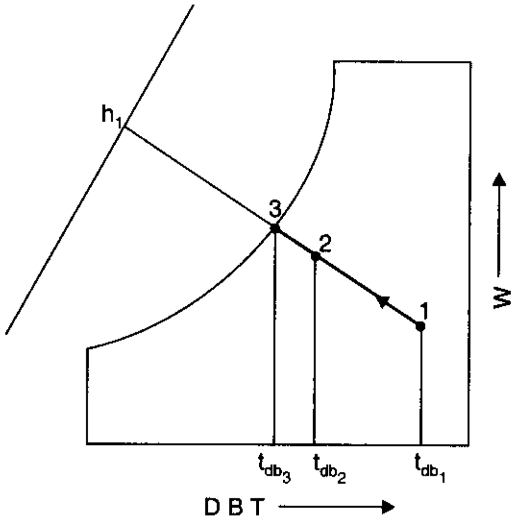


Fig. 10.13. Cooling and humidification.

ratio of dry-bulb temperature decrease to the entering wet bulb depression usually expressed as percentage. Thus, from Fig. 10.13, the saturating efficiency is :

$$\% \, \eta_{sat} = \left(\frac{t_{db_1} - t_{db_2}}{t_{db_1} - t_{db_3}} \right) \times 100 \qquad \text{...(10.26)}$$

As a fraction, it is equal to one minus the by pass factor for the process.

This adiabatic process, for all practical purposes, is line of constant *enthalpy*. The moisture added can be obtained from the increase in specific humidity.

10.6.6. Heating and Dehumidification

If air is passed over a solid absorbent surface or through a liquid absorbent spray simultaneous heating and dehumidification is accompanied. In either case the dehumidification results from adsorbent or absorbent having a lower water vapour pressure than air. Moisture is condensed out of the air, and consequently the latent heat of condensation is liberated, causing sensible heating of air. If these were the only energies involved, the process would be the inverse of the adiabatic saturation process. There is, however, an additional energy absorbed or liberated by the active material, termed the *heat of adsorption* or *absorption.* For the solid adsorbents used commercially, such as silica gel or activated alumina, and for the more common liquid absorbents, such as solutions of organic salts or inorganic compounds like ethylene, glycol, heat is involved and results in additional sensible heating. Thus the path lies above a constant wet bulb line on the psychrometric chart such as path 1-2 in Fig. 10.14.

10.6.7. Heating and Humidification

If air is passed through a humidifier which has heated water sprays instead of simply recirculated spray, the air is humidified and may be heated, cooled or unchanged in temperature. In such a process the *air increases in specific humidity and the* enthalpy, and the *dry bulb temperature will increase or decrease according to the initial temperature of the air and that of the spray.* If sufficient water is supplied relative to the mass flow of air, the air will approach saturation at water temperature. Examples of such processes are shown on Fig. 10.15.

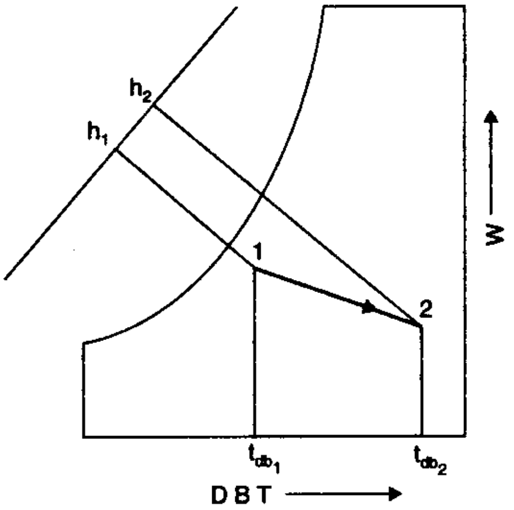


Fig. 10.14. Heating and dehumidification.

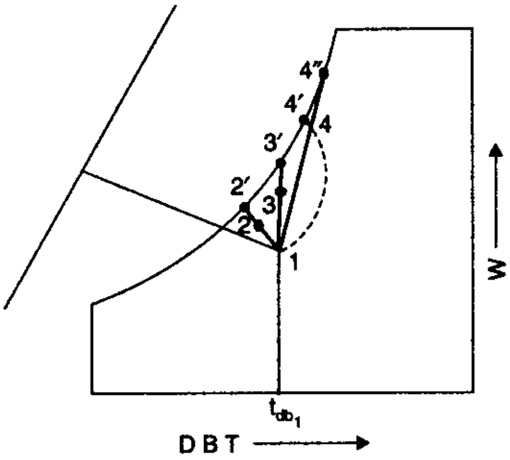


Fig. 10.15. Heating and humidification.

Process 1-2 : It denotes the cases in which the temperature of the heated spray water is less than the air DBT.

Process 1-3 : It denotes the cases in which the temperature is *equal to the air DBT.*

Process 1-4 : It denotes the cases in which a spray temperature is *greater than air DBT.*

As in the case of adiabatic saturation, the degree to which the process approaches saturation can be expressed in terms of the by-pass factor or a saturating efficiency.

If the water rate relative to the air quantity is smaller, the water temperature will drop significantly during the process. The resultant process will be a curved line such as the dashed 1-4 where 4 represents the leaving water temperature.

Note. It is possible to accomplish heating and humidification by evaporation from an open pan of heated water, or by direct injection of heated water or steam. The latter is more common. The process line for it is of little value because the process is essentially an instantaneous mixing of steam and the air. The final state point of the air can be found, however by making a humidity and enthalpy balance for the process. The solution of such a problem usually involves cut-and-try procedure.

Example 10.1. *The atmospheric conditions are ; 20°C and specific humidity of 0.0095 kg/kg of dry air. Calculate the following :*

(*i*) *Partial pressure of vapour* (*ii*) *Relative humidity*

(*iii*) *Dew point temperature.*

Solution. Dry bulb temperature, $t_{db} = 20°C$

Specific humidity, $W = 0.0095$ kg/kg of dry air

(*i*) **Partial pressure of vapour, p_v :**

The specific humidity is given by

$$W = \frac{0.622\, p_v}{p_t - p_v}$$

$$0.0095 = \frac{0.622\, p_v}{1.0132 - p_v}$$

$$0.0095(1.0132 - p_v) = 0.622\, p_v$$

$$0.009625 - 0.0095\, p_v = 0.622\, p_v$$

$$p_v = \mathbf{0.01524\ bar.} \quad \textbf{(Ans.)}$$

(*ii*) **Relative humidity** ϕ **:**

Corresponding to 20°C, from steam tables,

$$p_{vs} = 0.0234 \text{ bar}$$

∴ **Relative humidity,** $\phi = \frac{p_v}{p_{vs}} = \frac{0.01524}{0.0234} = \mathbf{0.65 \text{ or } 65\%.}$ **(Ans.)**

(*iii*) **Dew point temperature,** $\mathbf{t_{dp}}$ **:**

The dew point temperature is the saturation temperature of water vapour at a pressure of 0.01524 bar,

t_{dp} [from steam tables by interpolation]

$$= 13 + \frac{(14-13)}{(0.01598 - 0.0150)} \times [0.01524 - 0.0150]$$

$$= 13 + \frac{0.00024}{0.00098} = \mathbf{13.24°C.} \quad \textbf{(Ans.)}$$

Example 10.2. *The air supplied to a room of a building in winter is to be at 17°C and have a relative humidity of 60%. If the barometric pressure is 1.01325 bar, find : (i) The specific humidity ; (ii) The dew point under these conditions.*

Solution. Dry bulb temperature, t_{db} = 17°C

Relative humidity, ϕ = 60%

Barometric or total pressure, p_t = 1.01325 bar

Specific humidity, W :

Corresponding to 17°C, from steam tables,

$$p_{vs} = 0.0194 \text{ bar}$$

Also, $\phi = \frac{p_v}{p_{vs}}$

i.e., $0.6 = \frac{p_v}{0.0194}$

∴ $p_v = 0.6 \times 0.0194 = 0.01164$ bar.

Specific humidity, $W = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.01164}{1.01325 - 0.01164}$

$$= \mathbf{0.00723\ kg/kg\ of\ dry\ air.} \quad \textbf{(Ans.)}$$

Dew point temperature, $\mathbf{t_{dp}}$ **:**

If the air is cooled at constant pressure the vapour will begin to condense at the saturation temperature corresponding to 0.01164 bar. By interpolation from steam tables, the dew point temperature, t_{dp} is then

$$t_{dp} = 9 + (10 - 9) \times \frac{0.01164 - 0.01150}{0.01230 - 0.01150} = \mathbf{9.18°C.} \quad \textbf{(Ans.)}$$

Example 10.3. *0.004 kg of water vapour per kg of atmospheric air is removed and temperature of air after removing the water vapour becomes 20°C. Determine :*

(*i*) *Relative humidity* (*ii*) *Dew point temperature.*

Assume that condition of atmospheric air is 30°C and 55% R.H. and pressure is 1.0132 bar.

Solution. Corresponding to 30ºC, from steam tables, $p_{vs} = 0.0425$ bar

$\therefore$ Relative humidity (R.H.), $\phi = \dfrac{p_v}{p_{vs}}$

i.e.,
$$0.55 = \frac{p_v}{0.0425}$$

$\therefore$
$$p_v = 0.02337 \text{ bar.}$$

Also the specific humidity,

$$W = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.02337}{1.0132 - 0.02337} = 0.01468 \text{ kg/kg of dry air.}$$

The specific humidity after removing 0.004 kg of water vapour becomes,

$$0.01468 - 0.004 = 0.01068 \text{ kg/kg of dry air}$$

and the temperature t_{db} is given as 20ºC.

The partial pressure of water vapour, p_v, at this condition can be calculated as follows :

$$W = \frac{0.622\, p_v}{p_t - p_v}$$

$$0.01068 = \frac{0.622\, p_v}{1.0132 - p_v}$$

or,
$$0.01068\,(1.0132 - p_v) = 0.622\, p_v$$

or,
$$0.01082 - 0.01068\, p_v = 0.622\, p_v$$

$$0.6327\, p_v = 0.01082$$

$\therefore$
$$p_v = 0.0171 \text{ bar}$$

Corresponding to 20ºC, from steam tables, $p_{vs} = 0.0234$ bar.

(*i*) **Relative humidity,** $\phi = \dfrac{p_v}{p_{vs}} = \dfrac{0.0171}{0.0234}$ = **0.73 or 73%. (Ans.)**

(*ii*) **Dew point temperature, t_{dp} :**

Corresponding to 0.0171 bar, from steam tables, $\mathbf{t_{dp}}$ = **15°C. (Ans.)**

Example 10.4. *The sling psychrometer in a laboratory test recorded the following readings :*

Dry bulb temperature = 35°C

Wet bulb temperature = 25°C.

Calculate the following :

(*i*) *Specific humidity* (*ii*) *Relative humidity*

(*iii*) *Vapour density in air* (*iv*) *Dew point temperature*

(*v*) *Enthalpy of mixture per kg of dry air*

Take atmospheric pressure = 1.0132 bar.

Solution. For finding the partial pressure of vapour, using the equation :

$$p_v = (p_{vs})_{wb} - \frac{[p_t - (p_{vs})_{wb}](t_{db} - t_{wb})}{1527.4 - 1.3\, t_{wb}}$$

Corresponding to 25ºC (from steam tables),

$$(p_{vs})_{wb} = 0.0317 \text{ bar}$$

Substituting the values in the above equation, we get

$$p_v = 0.0317 - \frac{[1.0132 - 0.0317](35 - 25)}{1527.4 - 1.3 \times 25} = 0.0317 - 0.0065 = 0.0252 \text{ bar.}$$

(*i*) **Specific humidity,**

$$W = \frac{0.622\,p_v}{p_t - p_v} = \frac{0.622 \times 0.0252}{(1.0312 - 0.0252)} = \mathbf{0.01586\ kg/kg\ of\ dry\ air.\ (Ans.)}$$

(*ii*) **Relative humidity,** $\phi = \frac{p_v}{p_{vs}} = \frac{0.0252}{0.0563}$

[p_{vs} = 0.0563 bar corresponding to 35°C, from steam tables]

$= \mathbf{0.447\ or\ 44.7\%.}$ **(Ans.)**

(*iii*) **Vapour density :**

From characteristic gas equation

$$p_v V_v = m_v R_v T_v$$

$$p_v = \frac{m_v}{V_v} R_v T_v = \rho_v R_v T_v$$

$$\left[\text{where } \rho_v = \text{vapour density, } R_v = \frac{\text{Universal gas constant}}{\text{Molecular weight of H}_2\text{O}} = \frac{8314.3}{18}\right]$$

$$\therefore \quad 0.0252 \times 10^5 = \rho_v \times \frac{8314.3}{18} \times (273 + 35)$$

$$\therefore \quad \rho_v = \frac{0.0252 \times 10^5 \times 18}{8314.3 \times 308} = \mathbf{0.0177\ kg/m^3.}\ \ \textbf{(Ans.)}$$

(*iv*) **Dew point temperature, t_{dp} :**

Corresponding to 0.0252 bar, from steam tables (by interpolation),

$$t_{dp} = 21 + (22 - 21) \times \frac{(0.0252 - 0.0249)}{(0.0264 - 0.0249)} = \mathbf{21.2°C.}\ \ \textbf{(Ans.)}$$

(*v*) **Enthalpy of mixture per kg of dry air, h :**

$$h = c_p t_{db} + W h_{\text{vapour}}$$

$$= 1.005 \times 35 + 0.01586\,[h_g + 1.88\,(t_{db} - t_{dp})]$$

$$= 35.175 + 0.01586\,[2565.3 + 1.88\,(35 - 21.2)]$$

(where h_g = 2565.3 kJ/kg corresponding to 35°C t_{db})

$= \mathbf{76.27\ kJ/kg\ of\ dry\ air.}$ **(Ans.)**

Example 10.5. Adiabatic mixing : *One kg of air at 35°C DBT and 60% R.H. is mixed with 2 kg of air at 20°C DBT and 13°C dew point temperature. Calculate the specific humidity of the mixture.*

Solution. For the air at 35°C DBT and 60% R.H. :

Corresponding to 35°C, from steam tables,

$$p_{vs} = 0.0563 \text{ bar}$$

Relative humidity, $\phi = \frac{p_v}{p_{vs}}$

$$\therefore \quad p_v = \phi\, p_{vs} = 0.6 \times 0.0563 = 0.0338 \text{ bar}$$

$$W = \frac{0.622\,p_v}{p_t - p_v} = \frac{0.622 \times 0.0338}{1.0132 - 0.0338} = 0.0214 \text{ kg/kg of dry air}$$

Corresponding to 0.0338 bar, from steam tables,

$$t_{dp} = 26 + (27 - 26)\,\frac{(0.0338 - 0.0336)}{(0.0356 - 0.0336)} = 26.1°C$$

Enthalpy,

$$\begin{aligned} h &= c_p t_{db} + W h_{\text{vapour}} \\ &= 1.005\ t_{db} + W\,[h_g + 1.88\ (t_{db} - t_{dp})] \\ &= 1.005 \times 35 + 0.0214\ [2565.3 + 1.88\ (35 - 26.1)] \\ &= 90.43\ \text{kJ/kg of dry air.} \end{aligned}$$

For the air at 20°C DBT and 13°C dew point temperature :

p_v is the vapour pressure corresponding to the saturation pressure of steam at 13°C.

$$\therefore \qquad p_v = 0.0150\ \text{bar}$$

$$W = \frac{0.622\,p_v}{p_t - p_v} = \frac{0.622 \times 0.015}{1.0132 - 0.015} = 0.00935\ \text{kg/kg of dry air}$$

Enthalpy, $h = c_p t_{db} + W h_{\text{vapour}}$

$$\begin{aligned} &= 1.005 \times 20 + 0.00935\ [h_g + 1.88\ (t_{db} - t_{dp})] \\ &= 20.1 + 0.00935\ [2538.1 + 1.88\ (20 - 13)] \\ &= 43.95\ \text{kJ/kg of dry air} \end{aligned}$$

Now enthalpy per kg of moist air

$$= \frac{1}{3}\left[\frac{90.43}{1.0214} + \frac{43.95 \times 2}{1.00935}\right] = 58.54\ \text{kJ/kg of moist air}$$

Mass of vapour/kg of moist air

$$= \frac{1}{3}\left[\frac{0.0214}{1.0214} + \frac{0.00935 \times 2}{1.00935}\right] = 0.01316\ \text{kg/kg of moist air}$$

Specific humidity of mixture

$$= \frac{0.01316}{1 - 0.01316} = \mathbf{0.01333\ kg/kg\ of\ dry\ air.} \quad \textbf{(Ans.)}$$

Example 10.6. Sensible heating : *90 m³ of air per minute at 20°C and 75% R.H. is heated until its temperature becomes 30°C. Calculate :*

(i) R.H. of the heated air. *(ii) Heat added to air per minute.*

Solution. (*i*) ***For air at 20°C and 75% R.H. :***

$$p_{vs} = 0.0234\ \text{bar (from steam tables, at 20°C)}$$

$$p_v = \phi \times p_{vs} = 0.75 \times 0.0234 = 0.01755\ \text{bar}$$

$$t_{dp} = 15 + (16 - 15)\ \frac{(0.01755 - 0.017)}{(0.0182 - 0.017)} \simeq 15.5°\text{C}$$

$$W_1 = \frac{0.622\,p_v}{p_t - p_v} = \frac{0.622 \times 0.01755}{1.0132 - 0.01755} = 0.0109\ \text{kg/kg of dry air}$$

Enthalpy, $h_1 = c_p t_{db} + W h_{\text{vapour}}$

$$\begin{aligned} &= 1.005 \times 20 + 0.0109\ [h_g + 1.88\ (t_{db} - t_{dp})] \\ &= 1.005 \times 20 + 0.0109\ [2538.1 + 1.88(20 - 15.5)] = 47.85\ \text{kJ/kg of dry air} \end{aligned}$$

(*i*) **Relative humidity of heated air :**

For air at 30°C DBT :

Since the saturation pressure of water vapour at 30°C is higher than the saturation pressure of water vapour at 20°C so it is *sensible heating,* where p_v is same after heating.

$$\therefore \qquad \text{Relative humidity, } \phi = \frac{p_v}{p_{vs}} = \frac{0.01755}{0.0425} = 0.412 \text{ or } 41.2\%$$

(p_{vs} = 0.0425 bar, corresponding to 30°C)

i.e., Relative humidity of heated air = **41.2%. (Ans.)**

(*ii*) **Heat added to air per minute :**

Enthalpy, $h_2 = c_p t_{db} + W h_{vapour}$

$$= 1.005 \times 30 + 0.0109 \times [h_g + 1.88\,(t_{db} - t_{dp})]$$
$$= 1.005 \times 30 + 0.0109\,[2556.3 + 1.88\,(30 - 15.5)]$$
$$= 58.31 \text{ kJ/kg of dry air}$$

Mass of dry air in 90 m^3 of air supplied

$$m_a = \frac{pV}{RT} = \frac{(p_t - p_v)V}{RT}$$
$$= \frac{(1.0132 - 0.01755) \times 10^5 \times 90}{287 \times (273 + 20)} = 106.5 \text{ kg/min.}$$

Amount of heat added per minute

$$= 106.5\,(h_2 - h_1) = 106.5\,(58.31 - 47.85) \simeq \mathbf{1114\ kJ.}\ \ \textbf{(Ans.)}$$

Example 10.7. Sensible cooling : *40 m^3 of air at 35°C DBT and 50% R.H. is cooled to 25°C DBT maintaining its specific humidity constant. Determine :*

(i) Relative humidity (R.H.) of cooled air ;

(ii) Heat removed from air.

Solution. ***For air at 35°C DBT and 50% R.H. :***

$$p_{vs} = 0.0563 \text{ bar (At 35°C, from steam tables)}$$

$$\phi = \frac{p_v}{p_{vs}}$$

$$\therefore \quad p_v = \phi \times p_{vs} = 0.5 \times 0.0563 = 0.02815 \text{ bar}$$

$$W_1 = \frac{0.622\,p_v}{p_t - p_v} = \frac{0.622 \times 0.02815}{1.0132 - 0.02815} = 0.0177 \text{ kg/kg of dry air}$$

$$h_1 = c_p\,t_{db_1} + W_1\,[h_{g_1} + 1.88\,(t_{db_1} - t_{dp_1}]$$

$$t_{dp_1} \simeq 23°\text{C (corresponding to 0.02815 bar)}$$

$$\therefore \quad h_1 = 1.005 \times 35 + 0.0177\,[2565.3 + 1.88\,(35 - 23)] = 80.98 \text{ kJ/kg of dry air}$$

For air at 25°C DBT :

(*i*) ***R.H. of cooled air :***

Since the specific humidity remains constant the vapour pressure in the air remains constant.

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.02815}{0.0317} = 0.888 \text{ or } 88.8\%$$

i.e., Relative humidity of the cooled air = **88.8%. (Ans.)**

(*ii*) ***Heat removed from air :***

$$h_2 = c_p\,t_{db_2} + W_2\,[h_{g_2} + 1.88\,(t_{db_2} - t_{dp_2})]$$
$$= 1.005 \times 25 + 0.0177\,[2547.2 + 1.88\,(25 - 23)]$$
$$= 70.27 \text{ kJ/kg of dry air.}$$

To find mass of dry air (m_a), using the relation :

$$p_a v_a = m_a R_a T_a \qquad \left[\begin{array}{l} \because\ W_1 = W_2 = 0.0177 \text{ kg/kg of dry air} \\ t_{dp_2} = t_{dp_1} = 23°\text{C since } p_v \text{ does not change} \end{array}\right]$$

$$\therefore \quad m_a = \frac{p_a v_a}{R_a T_a} = \frac{(1.0132 - 0.02815) \times 10^5 \times 40}{287 \times (273 + 35)} = 44.57 \text{ kg}$$

∴ Heat removed from 40 m^3 of air

$$= m_a\,(h_1 - h_2) = 44.57\,(80.98 - 70.27) = \mathbf{477.3\ kJ.}\ \ \textbf{(Ans.)}$$

Example 10.8. Cooling and dehumidification : *120 m³ of air per minute at 35°C DBT and 50% relative humidity is cooled to 20°C DBT by passing through a cooling coil.*

Determine the following :

(*i*) *Relative humidity of out coming air and its wet bulb temperature.*

(*ii*) *Capacity of cooling coil in tonnes of refrigeration.*

(*iii*) *Amount of water vapour removed per hour.*

Solution. ***For the air at 35°C DBT and 50% R.H. :***

$$p_{vs} = 0.0563 \text{ bar (At 35°C, from steam tables)}$$

$$p_v = \phi \times p_{vs} = 0.5 \times 0.0563 = 0.02815 \text{ bar.}$$

$$W_1 = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.02815}{1.0132 - 0.02815} = 0.0177 \text{ kg/kg of dry air.}$$

$$h_1 = c_p\, t_{db_1} + W_1 [h_{g_1} + 1.88\,(t_{db_1} - t_{dp_1})]$$

$$t_{dp_1} \approx 23°\text{C (Corresponding to 0.02815 bar).}$$

$$\therefore \quad h_1 = 1.005 \times 35 + 0.0177\, [2565.3 + 1.88\,(35 - 23)]$$

$$= 80.98 \text{ kJ/kg of dry dir.}$$

For the air at 20°C. As the saturation vapour pressure at 20°C is 0.0234 bar, less than the vapour pressure 0.02815 bar at 35°C, so that *condensation takes place* and air *will be saturated at 20°C.*

(*i*) ∴ **Relative humidity of exit air is 100 per cent. (Ans.)**

Since the air is saturated, wet bulb temperature is equal to dry bulb temperature = 20°C. (Ans.)

$$\therefore \quad p_v = p_{vs} = 0.0234 \text{ bar.}$$

$$W_2 = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.0234}{(1.0132 - 0.0234)} = 0.0147 \text{ kg/kg of dry air}$$

$$h_2 = c_p\, t_{db_2} + W_2 [h_{g_2} + 1.88\,(t_{db_2} - t_{dp_2})]$$

$$= 1.005 \times 20 + 0.0147\, [2538.1 + 1.88\,(20 - 20)]$$

[∵ When air is saturated $t_{db} = t_{dp}$]

$$= 57.41 \text{ kJ/kg of dry air}$$

The weight of water vapour removed per kg of dry air

$$= 0.0177 - 0.0147 = 0.003 \text{ kg/kg of dry air}$$

Heat removed per kg of dry air

$$= h_1 - h_2 = 80.98 - 57.41 = 23.57 \text{ kJ/kg of dry air}$$

Mass of dry air passing per minute

$$m_a = \frac{p_a V_a}{R_a T_a} = \frac{(1.0132 - 0.02815) \times 10^5 \times 120}{287 \times (35 + 273)} = 133.7 \text{ kg/min}$$

(*ii*) **Capacity of the cooling coil in tonnes of refrigeration**

$$= \frac{m_a\,(h_1 - h_2)}{14000} = \frac{133.7 \times 23.57 \times 60}{14000} = \textbf{13.5 TR. (Ans.)}$$

(*iii*) **Amount of water removed per hour**

$$= m_a\,(W_1 - W_2) \times 60$$

$$= 133.7\,(0.0177 - 0.0147) \times 60 = \textbf{24.066 kg/h. (Ans.)}$$

Example 10.9. Adiabatic humidification : *150 m^3 of air per minute is passed through the adiabatic humidifier. The condition of air at inlet is 35°C DBT and 20 per cent relative humidity and the outlet condition is 20°C DBT and 15°C WBT.*

Determine the following :

(i) Dew point temperature *(ii) Relative humidity of the exit air*

(iii) Amount of water vapour added to the air per minute.

Solution. ***For air at 35°C DBT and 20% relative humidity.***

$$p_{vs} = 0.0563 \text{ bar (At 35°C from steam tables)}$$

$$p_v = \phi \times p_{vs} = 0.2 \times 0.0563 = 0.01126 \text{ bar}$$

$$W_1 = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.01126}{1.0132 - 0.01126} = 0.00699 \text{ kg/kg of dry air}$$

(*i*) The dew point temperature of air which is the saturation temperature of steam corresponding to the pressure 0.01126 bar is

$$8 + (9 - 8) \times \frac{(0.01126 - 0.01072)}{(0.01150 - 0.01072)} = 8.7°\text{C}$$

i.e., **Dew point temperature = 8.7°C. (Ans.)**

(*ii*) **Relative humidity of the exit air** :

For air at 20°C *DBT* and 15°C *WBT*.

$$p_v = (p_{vs})_{wb} - \frac{[p_t - (p_{vs})_{wb}](t_{db} - t_{wb})}{1527.4 - 1.3\, t_{wb}}$$

$$= 0.0170 - \frac{[1.0132 - 0.0170](20 - 15)}{1527.4 - 1.3 \times 15} = 0.0137 \text{ bar}$$

$$W_2 = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.0137}{(1.0132 - 0.0137)} = 0.00852 \text{ kg/kg of dry air}$$

Relative humidity $= \frac{p_v}{p_{vs}} = \frac{0.0137}{0.0234} = \mathbf{0.585 \text{ or } 58.5\%}$. **(Ans.)**

($\because$ p_{vs} = 0.0234 bar, corresponding to 20°C, from steam tables)

The *dew point temperature of air* which is the saturation temperature of steam corresponding to 0.0137 bar is **11°C** (from steam tables). **(Ans.)**

The amount of water vapour per kg of dry air

$$= W_2 - W_1 = 0.00852 - 0.00699 = 0.00153 \text{ kg}$$

The mass of dry air in 150 m^3 of mixture

$$m_a = \frac{p_a V_a}{R_a T_a} = \frac{(1.0132 - 0.01126) \times 10^5 \times 150}{287 \times (35 + 273)} = 170 \text{ kg}$$

(*iii*) **The amount of water vapour added to air per minute**

$$= m_a (W_2 - W_1) = 170 \times 0.00153 = \mathbf{0.26 \text{ kg/min.}} \quad \textbf{(Ans.)}$$

Example 10.10. Adiabatic saturation process : *An air-water vapour mixture enters an adiabatic saturation chamber at 28°C and leaves at 18°C, which is the adiabatic saturation temperature. The pressure remains constant at 1.0 bar.*

Determine the relative humidity and humidity ratio of the inlet mixture.

Solution. The specific humidity at the *exit*

$$W_{2s} = \frac{0.622\, p_s}{p_t - p_s} = \frac{0.622 \times 0.0206}{(1.00 - 0.0206)} = 0.01308 \text{ kg/kg of dry air}$$

The specific humidity at the inlet (equation 10.18)

$$W_1 = \frac{c_p\,(t_{db_2} - t_{db_1}) + W_{2s}\,(h_{g_2} - h_{f_2})}{h_{g_1} - h_{f_2}}$$

$$= \frac{1.005\,(18 - 28) + 0.01308\,(2534.4 - 75.6)}{(2552.6 - 75.6)}$$

$$= \frac{42.211}{2477} = 0.01704 \text{ kg/kg of dry air}$$

$$W_1 = \frac{0.622\,p_{v_1}}{p_t - p_{v_1}}$$

$$\therefore \quad 0.01704 = \frac{0.622\,p_{v_1}}{1.00 - p_{v_1}}$$

or $$0.01704\,(1.00 - p_{v_1}) = 0.622\;p_{v_1}$$

or $$0.01704 - 0.01704\;p_{v_1} = 0.622\;p_{v_1}$$

or $$0.0639\;p_{v_1} = 0.01704$$

$$\therefore \quad p_{v_1} = 0.02666 \text{ bar}$$

$\therefore$ **Relative humidity** $= \dfrac{p_{v_1}}{p_{s_1}} = \dfrac{0.02666}{0.03782}$ **= 0.7 or 70%. (Ans.)**

Example 10.11. *An air-water vapour mixture enters an air-conditioning unit at a pressure of 1.0 bar. 38°C DBT, and a relative humidity of 75%. The mass of dry air entering is 1 kg/s. The air-vapour mixture leaves the air-conditioning unit at 1.0 bar, 18°C, 85% relative humidity. The moisture condensed leaves at 18°C. Determine the heat transfer rate for the process.*

Solution. $t_{db_1} = 38°\text{C}$, R.H., $\phi_1 = 75\%$

$t_{tb_2} = 18°\text{C}$, R.H., $\phi_2 = 85\%$

The flow diagram and the process are shown in Figs. 10.16 (*a*) and (*b*) respectively.

At 38°C

From steam tables : $p_{vs} = 0.0663$ bar, $h_{g_1} = 2570.7$ kJ/kg

$$\therefore \quad p_v = \phi \times p_{vs} = 0.75 \times 0.0663 = 0.0497 \text{ bar}$$

$$W_1 = \frac{0.622 \times 0.0497}{1.0 - 0.0497} = 0.0325 \text{ kg/kg of dry air}$$

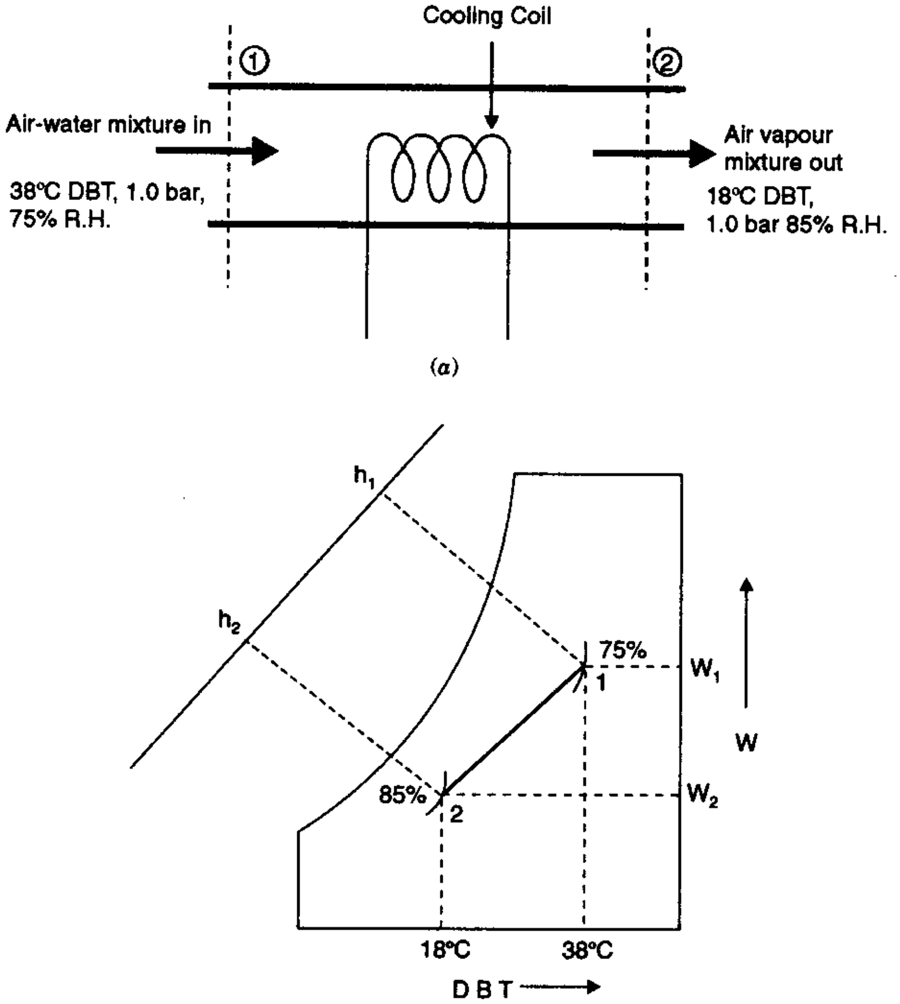
At 18°C

From steam tables : $p_{vs} = 0.0206$ bar, $h_{g_2} = 2534.4$ kJ/kg

$$h_{f_2} = 75.6 \text{ kJ/kg}$$

$$p_v = 0.85 \times 0.0206 = 0.01751 \text{ bar}$$

$$W_2 = \frac{0.622 \times 0.01751}{1 - 0.01751} = 0.01108 \text{ kg/kg of dry air}$$



(a)

(b)

Fig. 10.16

Heat transfer rate,

$$q = (W_2 h_{g_2} - W_1 h_{g_1}) + c_p (t_{db_2} - t_{db_1}) + (W_1 - W_2)\, h_{f_2}$$

$$= (0.01108 \times 2534.4 - 0.0325 \times 2570.7) + 1.005\,(18 - 38) + (0.0325 - 0.01108) \times 75.6$$

$$= -\,55.46 - 20.1 + 1.62 = -\,\mathbf{73.94\ kJ/kg\ of\ dry\ air.}$$ **(Ans.)**

Example 10.12. Evaporative Cooler : *Atmospheric air at 38°C and 25% relative humidity passes through an evaporator cooler. If the final temperature of air is 18°C, how much water is added per kg of dry air and what is the final relative humidity ?*

Solution. At 38°C :

$$p_{vs} = 0.0663 \text{ bar}, \quad h_{g_1} = 2570.7 \text{ kJ/kg}$$

and $$p_v = \phi \times p_{vs} = 0.25 \times 0.0663 = 0.01657 \text{ bar}$$

At 18°C :

$$h_{g_2} = 2534.4 \text{ kJ/kg}, \; p_{vs} = 0.0206 \text{ bar}$$

$$\therefore \quad W_1 = \frac{0.622 \times 0.01657}{1.0132 - 0.01657} = 0.01037 \text{ kg/kg of dry air}$$

Since enthalpy remains constant during the process

$$\therefore \quad c_p t_{db_1} + W_1 h_{g_1} = c_p t_{db_2} + W_2 h_{g_2}$$

$$1.005 \times 38 + 0.01034 \times 2570.7 = 1.005 \times 18 + W_2 \times 2534.4$$

($\because$ At 18°C, $h_{g_2} = 2534.4$ kJ/kg)

i.e.,

$$W_2 = \frac{1.005 \times 38 + 0.01034 \times 2570.7 - 1.005 \times 18}{2534.4}$$

$$= 0.01842 \text{ kg/kg of dry air}$$

$\therefore$ **Amount of water added** $= W_2 - W_1 = 0.01842 - 0.01034$

$= \mathbf{0.00808}$ **kg/kg of dry air. (Ans.)**

Also,

$$0.00808 = \frac{0.622 \, p_{v_2}}{1.0132 - p_{v_2}}$$

or

$$0.00808 \, (1.0132 - p_{v_2}) = 0.622 \, p_{v_2}$$

$$0.00818 - 0.00808 \, p_{v_2} = 0.622 \, p_{v_2}$$

$$\therefore \quad p_{v_2} = 0.01298 \text{ bar}$$

$\therefore$ **Final relative humidity** $= \dfrac{0.01298}{0.0206} =$ **0.63 or 63%. (Ans.)**

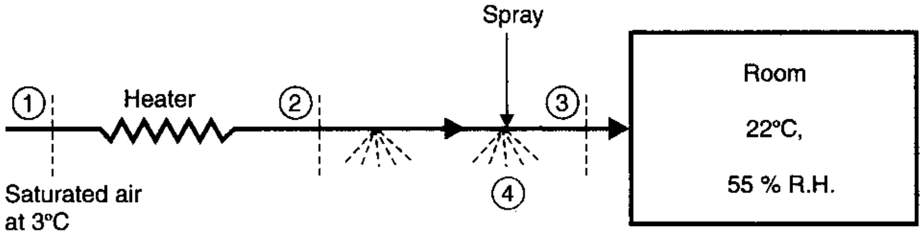
Example 10.13. *Saturated air at 3°C is required to be supplied to a room where the temperature must be held at 22°C with a relative humidity of 55%. The air is heated and then water at 10°C is sprayed to give the required humidity. Determine :*

(*i*) *The mass of spray water required per m³ of air at room conditions.*

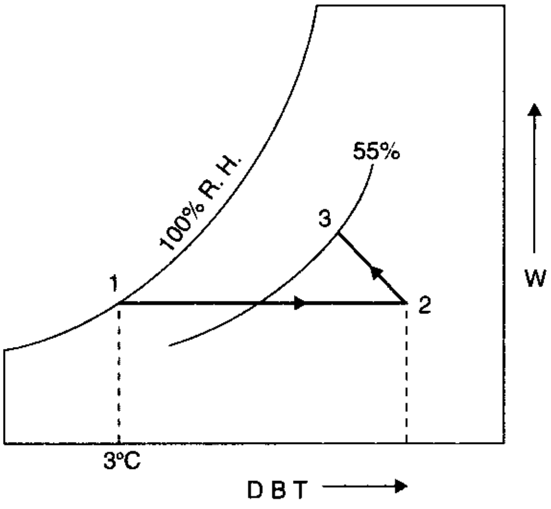
(*ii*) *The temperature to which the air must be heated.*

Neglect the fan power. Assume that the total pressure is constant at 1.0132 bar.

Solution. (*i*) The flow diagram is shown in Fig. 10.17 (*a*) and the processes are shown in Fig. 10.17 (*b*).



(*a*)



(*b*)

Fig. 10.17

(*i*) **Mass of spray water required**

At 22°C

From steam tables : $p_{vs} = 0.0264$ bar

$$\phi_3 = \frac{p_{v_3}}{p_{vs_3}} = \frac{p_{vs_3}}{0.0264} = 0.55$$

$$\therefore \quad p_{v_3} = 0.55 \times 0.0264 = 0.01452 \text{ bar}$$

$$\therefore \quad W_3 = \frac{0.622 \times 0.01452}{(1.0132 - 0.01452)} = 0.00904 \text{ kg/kg of dry air}$$

At 3°C

From steam tables : $p_{vs} = 0.0076$ bar

$$\phi_1 = \frac{p_{v_1}}{p_{vs_1}} = 1.00$$

$$\therefore \quad p_{v_1} = p_{vs_1} = 0.0076 \text{ bar}$$

$$W_1 = \frac{0.622 \times 0.0076}{1.0132 - 0.0076} = 0.0047 \text{ kg/kg of dry air}$$

$$W_3 - W_1 = 0.00904 - 0.0047 = 0.00434 \text{ kg/kg of dry air}$$

$$v_{a_3} = \frac{R_a T_3}{p_{a_3}} = \frac{287 \times (273 + 22)}{(1.0132 - p_{v_3}) \times 10^5} = \frac{287 \times 295}{0.9987 \times 10^5}$$

$$= 0.847 \text{ m}^3\text{/kg of dry air}$$

Spray water = $\frac{0.00434}{0.847}$ = **0.005124 kg moisture/m³. (Ans.)**

(*ii*) **Temperature to which the air must be heated t_{db_2} :**

Now $\quad h_2 + (W_3 - W_2)\, h_4 = h_3$

$$[c_p t_{db_2} + W_2 h_{\text{vapour (2)}}] + (W_3 - W_2)h_2 = c_p t_{db_3} + W_3 h_{\text{vapour (3)}}$$

$$\therefore \quad c_p(t_{db_3} - t_{db_2}) + W_3 h_{\text{vapour (3)}} - W_2 h_{\text{vapour (2)}} - (W_3 - W_2)h_4 = 0$$

From the steam tables at $p_v = 0.01452$ bar : $h_g = 2524$ kJ/kg

and $$t_{dp} = t_{sat} = 12.5°\text{C}$$

$$\therefore \quad 1.005(22 - t_{db_2}) + 0.00904[2524 + 1.88(22 - 12.5)]$$

$$- 0.0047\,[2524 + 1.88(t_{db_2} - 12.5)] - (0.00434 \times 4.187 \times 10) = 0$$

$$22.11 - 1.005\, t_{db_2} + 22.97 - 11.86 - 0.0088\, t_{db_2} + 0.11 - 0.1817 = 0$$

$$1.014\, t_{db_2} = 33.148$$

$$\therefore \quad \mathbf{t_{db_2} = 32.7°C.} \quad \textbf{(Ans.)}$$

Example 10.14. Cooling tower : *A small-size cooling tower is designed to cool 5.5 litres of water per second, the inlet temperature of which is 44°C. The motor-driven fan induces 9 m³/s of air through the tower and the power absorbed is 4.75 kW. The air entering the tower is at 18°C, and has a relative humidity of 60%. The air leaving the tower can be assumed to be saturated and its temperature is 26°C. Calculate :*

(i) The amount of cooling water (make-up) required per second.

(ii) The final temperature of the water.

Assume that the pressure remains constant throughout the tower at 1.013 bar.

Solution. The cooling tower is shown diagrammatically in Fig. 10.18.

(*i*) **Make-up water required :**

At 18°C $$p_{vs} = 0.0206 \text{ bar},$$

$$\therefore \quad p_v = \phi \times p_{vs} = 0.6 \times 0.0206 = 0.01236 \text{ bar}$$

$$\therefore \quad p_{a_1} = 1.013 - 0.01236 = 1.00064 \text{ bar}$$

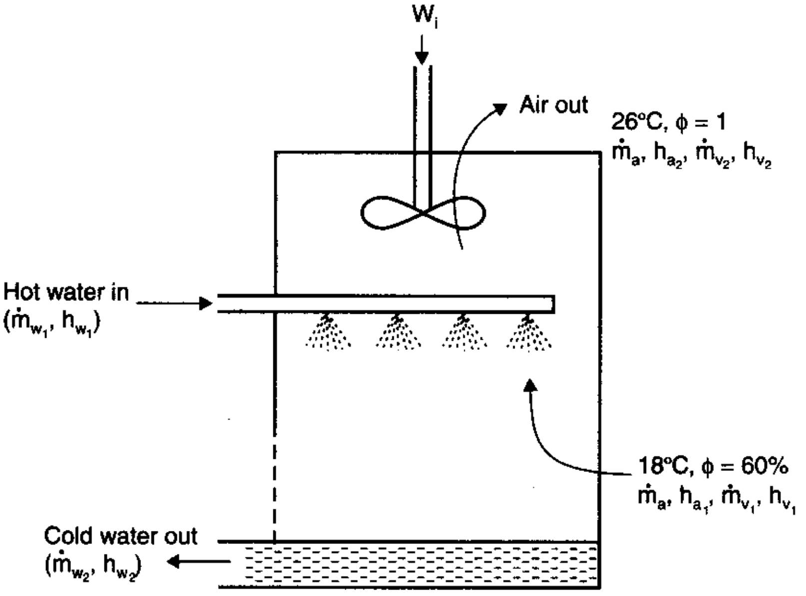


Fig. 10.18

Then, $\dot{m}_a = \dfrac{10^5 \times 1.00064 \times 9}{(0.287 \times 10^3) \times (18 + 273)} = 10.78$ kg/s

and $\dot{m}_{v_1} = \dfrac{10^5 \times 0.01236 \times 9}{(0.4618 \times 10^3) \times (18 + 273)} = 0.0828$ kg/s

(Script '*v*' denotes vapour and the script '*a*' denotes the air).

At exit at 26°C, $p_{vs} = 0.0336$ bar and $\phi = 1$

$\therefore$ $p_v = p_{vs} = 0.0336$ bar

$$W_2 = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.0336}{1.013 - 0.0336} = 0.02133 \text{ kg.}$$

But $W = \dfrac{m_v}{m_a}$

$\therefore$ $\dot{m}_{v_2} = 0.02133 \times 10.78 = 0.23$ kg/s

Hence, **make-up water required**

$= 0.23 - 0.0828 =$ **0.1472 kg/s. (Ans.)**

(*ii*) **Final temperature of the water :**

Also, $\dot{m}_{w_1} = 5.5 \times 1 = 5.5$ kg/s

and $\dot{m}_{w_2} = \dot{m}_{w_1}$ – (make-up water)

$= 5.5 - 0.1472 = 5.353$ kg/s

Applying the steady flow energy equation and neglecting changes in kinetic energy and potential energy, we have

$$W_i + \dot{m}_{w_1} h_{w_1} + \dot{m}_{a_1} h_{a_1} + \dot{m}_{v_1} h_{v_1} = \dot{m}_{a_2} h_{a_2} + \dot{m}_{v_2} h_{v_2} + \dot{m}_{w_2} h_{w_2}$$

Now, W_i (*i.e.*, work input) = 4.75 kW = 4.75 kJ/s.

Evaluating the enthalpies from a datum of 0°C, we have :

$h_{w_1} = h_f$ at 44°C = 184.3 kJ/kg,

$h_{a_1} = 1.005\,(18 - 0) = 18.09$ kJ/kg,

$h_{v_1} = 2519.7 + 1.88\,(18 - 10) = 2534.74$ kJ/kg.

[Corresponding to $p_v = 0.01236$ bar, $t_s = t_{dp} \simeq 10$°C *i.e.*, the vapour is superheated]

$h_{v_1} = h_g$ at 26°C = 2549 kJ/kg

$h_{a_2} = 1.005\,(26 - 0) = 26.13$ kJ/kg.

Then, substituting, we get

$4.75 + 5.5 \times 184.3 + 10.78 \times 18.09 + 0.0828 \times 2534.74$

$= 10.78 \times 26.13 + 0.23 \times 2549 + 5.353 \times h_{w_2}$

or $5.353\, h_{w_2} = 1423.28 - 867.95 = 555.33$

or $h_{w_2} = 103.74$ kJ/kg.

By interpolation, $h_f = 103.74$ kJ/kg at 26.7°C.

Hence, **final temperature of water = 26.7°C. (Ans.)**

Example 10.15. *A cooling tower used in power plant consists of 10 big fans, $\dot{m}_{water}$ = 1000 kg/min. It is cooled from 35°C to 30°C. Atmospheric conditions are 35°C DBT, 25°C WBT. Air leaves the tower at 30°C, 90% RH. Find out the quantity of air handled per fan hour and the quantity of make-up water required per hour.* **(AMIE Winter, 1999)**

Solution. Refer Fig. 10.19.

Heat absorbed from the cooling tower

$$= \dot{m}_{\text{water}} \times c \times \Delta T$$
$$= (1000 \times 60) \times 4.186 \times (35 - 30)$$
$$= 1.256 \times 10^6 \text{ kJ/h}$$

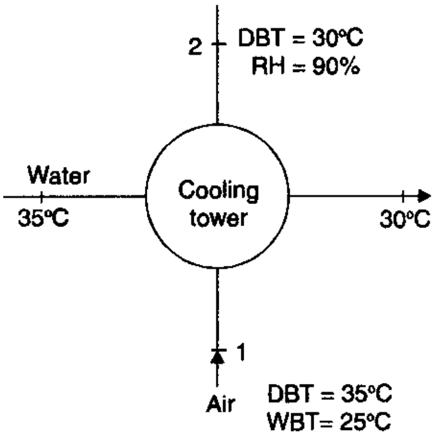


Fig. 10.19

From psychrometric chart, we have

At 35°C DBT and 25°C WBT :

h_1 = 76.5 kJ/kg ; W_1 = 0.016 kg/kg of air

At 30°C and 90% RH :

h_2 = 92.5 kJ/kg ; W_2 = 0.0246 kg/kg of air

Heat gained by air = Heat lost by water

$$\dot{m}_{\text{air}}\ (h_2 - h_1) = 1.256 \times 10^6$$

$$\therefore \quad \text{Mass of air, } \dot{m}_{\text{air}} = \frac{1.256 \times 10^6}{(h_2 - h_1)}$$

$$= \frac{1.256 \times 10^6}{(92.5 - 76.5)} = 78.5 \times 10^3 \text{ kg/h}$$

$\therefore$ **Quantity of air handled per fan** = $\dfrac{78.5 \times 10^3}{10}$ = **7850 kg/h. (Ans.)**

Quantity of make-up water = $\dot{m}_{air}\ (W_2 - W_1)$

= $78.5 \times 10^3\ (0.0246 - 0.016)$ = **675.1 kg/h. (Ans.)**

SOLUTIONS USING PSYCHROMETRIC CHARTS

Example 10.16. *The following data pertain to an air-conditioning system :*

Unconditioned space DBT = 30°C

Unconditioned space WBT = 22°C

Cold air duct supply surface temperature = 14°C.

Determine : (i) Dew point temperature.

(ii) Whether or not condensation will form on the duct.

Solution. Refer Fig. 10.20.

(*i*) To determine the dew point temperature for the given conditions, find the intersection of 30°C *DBT* and 22°C *WBT* and move horizontally (as shown by the arrow) to the dew point temperature scale. The **dew point (t_{dp}) is 18.6°C. (Ans.)**

(*ii*) Since the duct temperature (14°C) is less than t_{dp} (18.6°C) therefore *moisture will condense on the duct surface.* **(Ans.)**

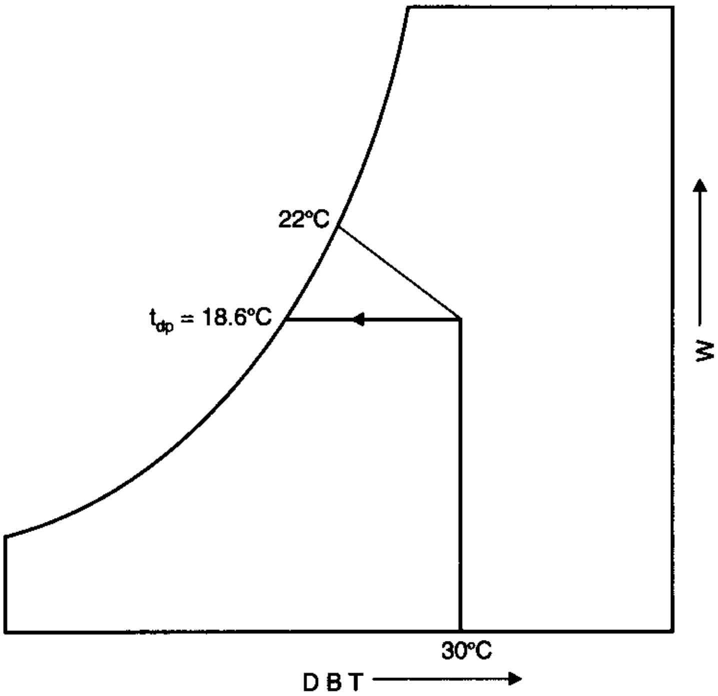


Fig. 10.20

Example 10.17. *200 m³ of air per minute at 15°C DBT and 75% R.H. is heated until its temperature is 25°C.*

Find : (i) R.H. of heated air.

(ii) Wet bulb temperature of heated air.

(iii) Heat added to air per minute.

Solution. Refer Fig. 10.21.

- Locate point 1 on the psychrometric chart on intersection of 15°C *DBT* and 75% R.H. lines.
- Through point 1 draw a horizontal line to cut 25°C *DBT* line and get point 2.
- Read the following values from the psychrometric chart :

$$h_1 = 35.4 \text{ kJ/kg}$$

$$h_2 = 45.2 \text{ kJ/kg}$$

$$v_{s_1} = 0.8267 \text{ m}^3\text{/kg.}$$

(*i*) **R.H. of heated air** (read from chart) = **41%. (Ans.)**

(*ii*) **WBT of heated air** (read from chart) = **16.1°C. (Ans.)**

(*iii*) Mass of air circulated per min., $m_a = \dfrac{200}{0.8267} = 241.9$ kg.

∴ **Heat added to air/min.**

$$= m_a (h_2 - h_1) = 241.9 (45.2 - 35.4) = \mathbf{2370.6 \text{ kJ}}. \quad \textbf{(Ans.)}$$

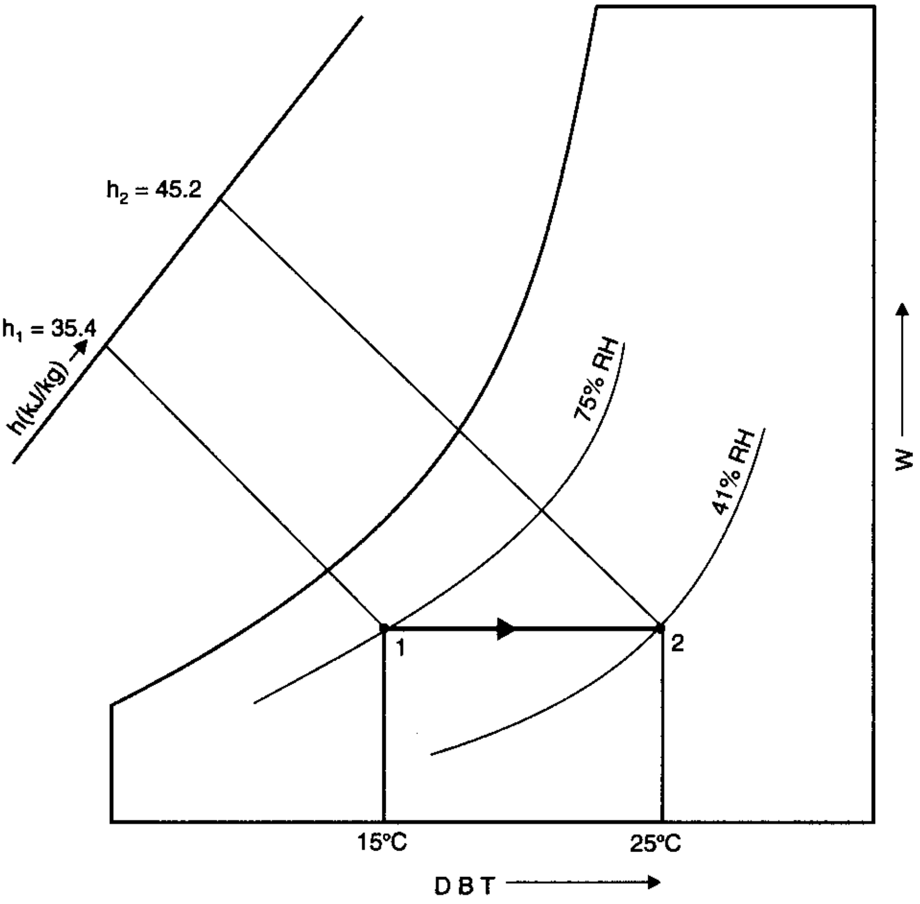


Fig. 10.21

Example 10.18. *It is required to design an air-conditioning plant for a small office room for following winter conditions :*

Outdoor conditions ...... 14°C DBT and 10°C WBT

Required conditions ...... 20°C DBT and 60% R.H.

Amount of air circulation ...... 0.30 m^3/min./person.

Seating capacity of office ...... 60.

The required condition is achieved first by heating and then by adiabatic humidifying.

Determine the following :

(*i*) *Heating capacity of the coil in kW and the surface temperature required if the by pass factor of coil is 0.4.*

(*ii*) *The capacity of the humidifier.*

Solve the problem by using psychrometric chart.

Solution. Refer Fig. 10.22.

- Locate the points '1' and '3' on the psychrometric chart.
- Draw a constant enthalpy line through '3' and constant specific humidity line through '1'.

- Locate the point '2' where the above two lines intersect.

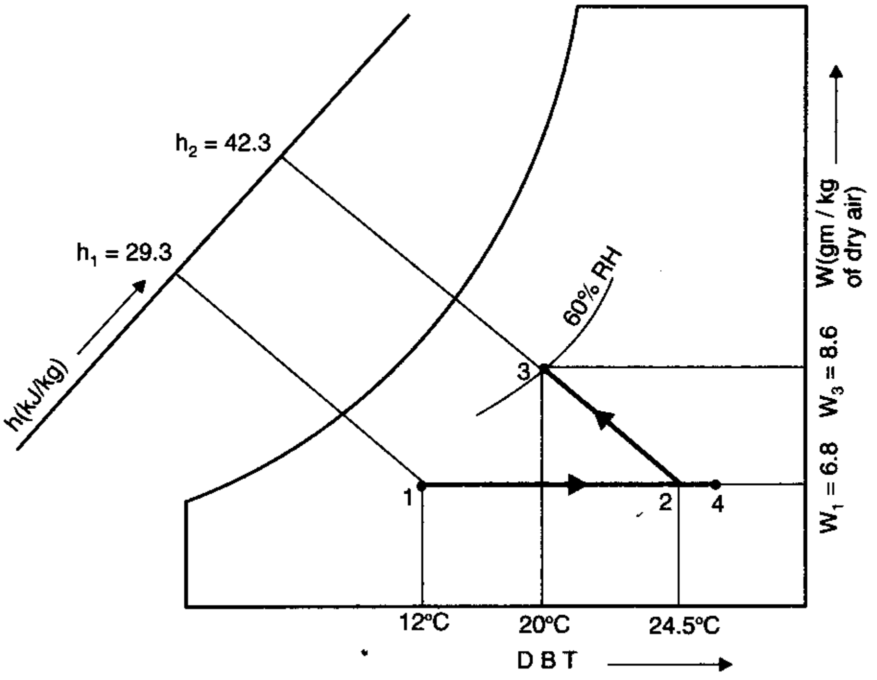


Fig. 10.22

From the psychrometric chart :

$$h_1 = 29.3 \text{ kJ/kg}, \; h_2 = h_3 = 42.3 \text{ kJ/kg}$$

$$t_{db_2} = 24.5°\text{C}, \; v_{s_1} = 0.817 \text{ m}^3/\text{kg}$$

The mass of air circulated per minute,

$$m_a = \frac{0.30 \times 60}{0.817} = 22.03 \text{ kg/min.}$$

(*i*) **Heating capacity of the heating coil**

$$= m_a(h_2 - h_1) = 22.03\,(42.3 - 29.3) = 286.4 \text{ kJ/min.}$$

$$= \mathbf{4.77 \text{ kJ/s or } 4.77 \text{ kW.}} \quad \textbf{(Ans.)}$$

The by-pass factor (*BF*) of heating coil is given by :

$$BF = \frac{t_{db_4} - t_{db_2}}{t_{db_4} - t_{db_1}}$$

$$0.4 = \frac{t_{db_4} - 24.5}{t_{db_4} - 12}$$

$$\therefore \quad 0.4\,(t_{db_4} - 12) = t_{db_4} - 24.5$$

i.e., t_{db_4} *(coil surface temperature)* = **32.8°C. (Ans.)**

(*ii*) **The capacity of the humidifier**

$$= \frac{m_a\,(W_3 - W_1)}{1000} \times 60 \text{ kg/h} = \frac{22.03\,(8.6 - 6.8)}{1000} \times 60 = \mathbf{2.379 \text{ kg/h.}} \quad \textbf{(Ans.)}$$

Example 10.19. *It is required to design an air-conditioning system for an industrial process for the following hot and wet summer conditions :*

Outdoor conditions ...... 32°C DBT and 65% R.H.

Required air inlet conditions ...... 25°C DBT and 60% R.H.

Amount of free air circulated ...... 250 m³/min.

Coil dew temperature ...... 13°C.

The required condition is achieved by first cooling and dehumidifying and then by heating.

Calculate the following :

(i) The cooling capacity of the cooling coil and its by-pass factor.

(ii) Heating capacity of the heating coil in kW and surface temperature of the heating coil if the by-pass factor is 0.3.

(iii) The mass of water vapour removed per hour.

Solve this problem with the use of psychrometric chart.

Solution. Refer Fig. 10.23.

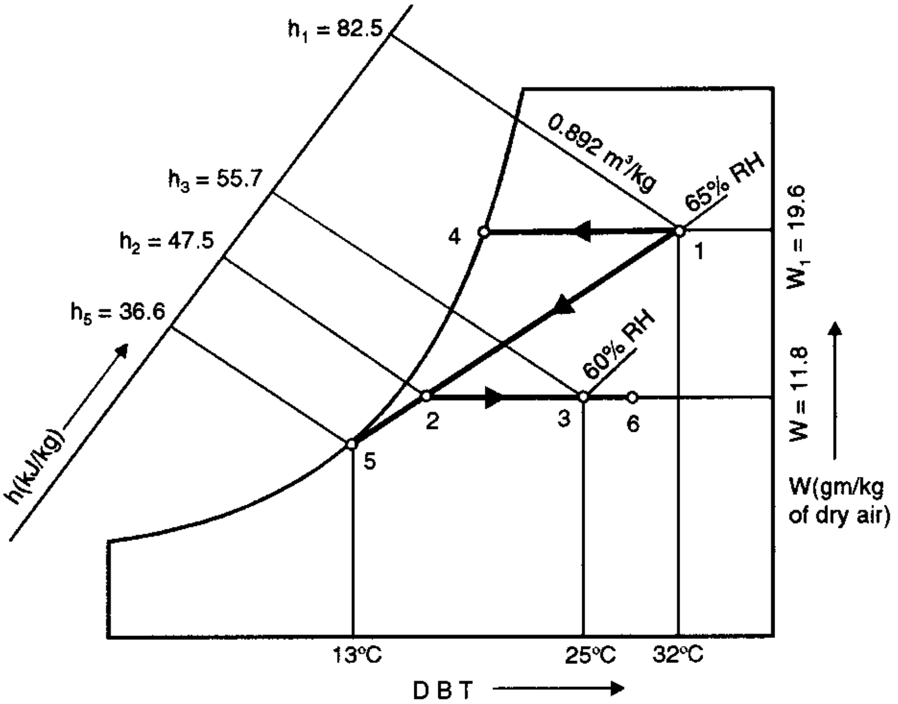


Fig. 10.23

- Locate the points '1', '5' and '3' as shown on psychrometric chart.
- Join the line 1-5.
- Draw constant specific humidity line through '3' which cuts the line 1-5 at point '2'. The point '2' is located in this way.

From psychrometric chart :

$$h_1 = 82.5 \text{ kJ/kg}, \qquad h_2 = 47.5 \text{ kJ/kg}$$
$$h_3 = 55.7 \text{ kJ/kg}, \qquad h_5 = 36.6 \text{ kJ/kg}$$

$$W_1 = 19.6 \text{ gm/kg}, \qquad W_3 = 11.8 \text{ gm/kg}$$

$$t_{db_2} = 17.6°C, \qquad v_{s_1} = 0.892 \text{ m}^3/\text{kg}.$$

The mass of air supplied per minute,

$$m_a = \frac{250}{0.892} = 280.26 \text{ kg/min.}$$

(*i*) **The capacity of the cooling coil**

$$= \frac{m_a (h_1 - h_2) \times 60}{14000} = \frac{280.26 (82.5 - 47.5) \times 60}{14000} = \mathbf{42.04 \text{ TR.}} \quad \textbf{(Ans.)}$$

The *by-pass factor of the cooling coil* is given by :

$$BF = \frac{h_2 - h_5}{h_1 - h_5} = \frac{47.5 - 36.6}{82.5 - 36.6} = \mathbf{0.237.} \quad \textbf{(Ans.)}$$

(*ii*) **The heating capacity of the heating coil**

$$= m_a (h_3 - h_2) = 280.26 (55.7 - 47.5) = 2298.13 \text{ kJ/min} = \frac{2298.13}{60} \text{ kJ/s}$$

$$= \mathbf{38.3 \text{ kW.}} \quad \textbf{(Ans.)}$$

The by-pass factor of the *heating coil* is given by

$$BF = \frac{t_{db_6} - t_{db_3}}{t_{db_6} - t_{db_2}}$$

$$0.3 = \frac{t_{db_6} - 25}{t_{db_6} - 17.6}$$

$$\therefore \quad t_{db_6} = 28.2°C.$$

Hence surface temperature of heating coil = 28.2°C. (Ans.)

(*iii*) **The mass of water vapour removed per hour**

$$= \frac{280.26 (W_1 - W_3) \times 60}{1000} = \frac{280.26 (19.6 - 11.8)}{1000} \times 60 = \mathbf{131.16 \text{ kg/h.}} \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. 'Air-conditioning' is the simultaneous control of temperature, humidity, motion and purity of the atmosphere in a confined space.
2. 'Psychrometry' is an art of measuring moisture content of air.

   The science which investigates the thermal properties of moist air, considers the measurement and control of the moisture content of air, and studies the effects of atmospheric moisture on material and human comfort may properly be termed 'Psychrometrics'.
3. When air is saturated *DBT, WBT, DPT* are equal.
4. *Dalton's law of partial pressure* states, that the total pressure of a mixture of gases is equal to the sum of partial pressure which the component gases would exert if each existed alone in the mixture volume at the mixture temperature.
5. Specific humidity, $W = \frac{0.622\, p_v}{p_t - p_v}$.
6. Degree of saturation $(\mu) = \frac{\text{Mass of water vapour associated with unit mass of dry air } (W)}{\text{Mass of water vapour associated with saturated unit mass of dry air } (W_s)}$.

7. Relative humidity, $\phi = \frac{p_v}{p_{vs}}$.
8. A 'Psychrometer' is a device which is used for measuring dry bulb and wet-bulb temperatures simultaneously.
9. The processes which affect the psychrometric properties of air are called *'psychrometric processes'*.

   Important psychrometric processes are :

   (*i*) Mixing of air streams
   (*ii*) Sensible heating
   (*iii*) Sensible cooling
   (*iv*) Cooling and dehumidification
   (*v*) Cooling and humidification
   (*vi*) Heating and dehumidification
   (*vii*) Heating and humidification.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. In an unsaturated air the state of a vapour is
   (*a*) wet
   (*b*) superheated
   (*c*) saturated
   (*d*) unsaturated.
2. For saturated air
   (*a*) Wet bulb depression is zero
   (*b*) Wet bulb depression is positive
   (*c*) Wet bulb depression is negative
   (*d*) Wet bulb depression can be either positive or negative.
3. Which one of the following statements is *correct* ?
   (*a*) Dew point temperature can be measured with the help of thermometer
   (*b*) Dew point temperature is the saturation temperature corresponding to the partial pressure of the water vapour in moist air.
   (*c*) Dew point temperature is the same as the thermodynamic wet bulb temperature.
   (*d*) For saturated air, dew point temperature is less than the wet bulb temperature.
4. During sensible heating of moist air, enthalpy
   (*a*) increases
   (*b*) decreases
   (*c*) remains constant
   (*d*) none of the above.
5. During sensible cooling, wet bulb temperature
   (*a*) decreases
   (*b*) increases
   (*c*) remains constant
   (*d*) can decrease or increase.
6. Which one of the following statements is *correct* ?
   (*a*) Evaporative cooling and sensible cooling is the same
   (*b*) Evaporative cooling is a cooling and humidification process
   (*c*) Evaporative cooling is a cooling and dehumidification process
   (*d*) Evaporative cooling is not effective for hot and dry climates.
7. An air washer can work as a
   (*a*) filter only
   (*b*) humidifier only
   (*c*) dehumidifier only
   (*d*) all of the above.
8. The relative humidity, during sensible heating,
   (*a*) can increase or decrease
   (*b*) increases
   (*c*) decreases
   (*d*) remains constant.
9. The vapour pressure, during sensible heating of moist air,
   (*a*) increases
   (*b*) decreases
   (*c*) can increase or decrease
   (*d*) remains constant.

10. The relative humidity, during heating and humidification,
(*a*) increases (*b*) decreases
(*c*) may increase or decrease (*d*) remains constant.

11. The relative humidity, during cooling and dehumidification of moist air
(*a*) increases (*b*) decreases
(*c*) can increase or decrease (*d*) remains constant.

ANSWERS

1. (*b*) **2.** (*a*) **3.** (*b*) **4.** (*a*) **5.** (*a*) **6.** (*b*) **7.** (*d*)
8. (*b*) **9.** (*d*) **10.** (*a*) **11.** (*c*).

THEORETICAL QUESTIONS

1. Define the following terms :
   (*i*) Saturated air (*ii*) Dry bulb temperature
   (*iii*) Dew point temperature (*iv*) Relative humidity
   (*v*) Specific humidity.
2. State 'Dalton's law of partial pressure'.
3. Derive the following relations :
   (*i*) Specific humidity, $W = 0.622 \dfrac{p_v}{p_t - p_v}$. (*ii*) Degree of saturation, $\mu = \dfrac{p_v}{p_s} \left[\dfrac{\left(1 - \dfrac{p_s}{p_t}\right)}{\left(1 - \dfrac{p_v}{p_t}\right)} \right]$.
4. Explain briefly with a neat sketch a 'sling psychrometer'.
5. Describe briefly any two of the following processes :
   (*i*) Sensible heating (*ii*) Cooling and dehumidification
   (*iii*) Heating and humidification (*iv*) Heating and dehumidification.
6. Write a short note on 'by-pass factor'.

UNSOLVED EXAMPLES

1. The atmospheric conditions are 30°C and specific humidity of 0.0215 kg/kg of air. Determine :
   (*i*) Partial pressure of air (*ii*) Relative humidity
   (*iii*) Dew point temperature.
   Atmospheric pressure = 756 mm Hg. [**Ans.** (*i*) 14.89 mm of Hg, (*ii*) 46.8%, (*iii*) 17°C]
2. A mixture of air and water vapour at 1 bar and 25°C has a dew point temperature of 15°C. Determine the relative humidity and specific humidity. [**Ans.** 53.8%, 0.01078 kg/kg of dry air]
3. An air-water vapour mixture at 1.24 bar has a temperature of 38°C and relative humidity of 60%. Calculate the kg-mass of water vapour per kg of air and per kg of mixture. Also find the dew point.
   [**Ans.** 0.0203 kg/kg of dry air, 0.0199 kg/kg of mixture, 28.5°C]
4. In a house, the temperature of the windows on a day in winter is 5°C. When the temperature in the room is 23°C, and the barometric pressure is 748.8 mm Hg, what would be maximum relative humidity that could be maintained in the room without condensation on the window panes ? Under these conditions, find the partial pressure of the water vapour and air, the specific humidity, and the density of the mixture.
   [**Ans.** 30.7%, 0.00872 bar, 0.989 bar, 0.00548 kg/kg of dry air, 1.164 kg/m^3]
5. Atmospheric air enters a heater at 4.5°C and 60% relative humidity and leaves at a temperature of 21°C. Calculate :

(*i*) The heat supplied to the air ;

(*ii*) The final relative humidity. [**Ans.** 16.8 kJ/kg, 29.2%]

6. The air supplied to a room of building in winter is to be at 17°C and have a relative humidity of 60%. If the barometric pressure is 1.01325 bar, calculate the specific humidity. What would be the dew point under these conditions ? [**Ans.** 0.00723 kg/kg of dry air, 9.18°C]

7. If air at the condition of example 6, is passed at the rate of 0.5 m^3/s over a cooling coil which is at a temperature of 6°C, calculate the amount of vapour which will be condensed. Assume that the barometric pressure is the same as in example 6, and that the air leaving the coil is saturated. [**Ans.** 3.082 kg/h]

8. An air and water vapour mixture at 1 bar and 26.7°C has a specific humidity of 0.0085. Determine the percentage saturation. [**Ans.** 37.7%]

9. A mixture of air and water vapour at 1.013 bar and 16°C has a dew point of 5°C. Determine the relative and specific humidities. [**Ans.** 48%, 0.0054 kg/kg of dry air]

10. Atmospheric air at a pressure of 760 mm Hg has a temperature of 32°C and a percentage saturation as determined from a psychrometric chart of 52%. Calculate

    (*i*) The partial pressure of the vapour and the dry air

    (*ii*) The specific humidity

    (*iii*) The dew point

    (*iv*) The density of the mixture.

    [**Ans.** (*i*) 0.0247 bar, 0.988 bar, (*ii*) 0.01556, (*iii*) 20.9°C, (*iv*) 1.147 kg/m^3]

11. In a laboratory test, a psychrometer recorded 36°C DBT and 30°C WBT. Calculate :

    (*i*) Vapour pressure (*ii*) Relative humidity (*iii*) Specific humidity

    (*iv*) Degree of saturation (*v*) Dew point temperature (*vi*) Enthalpy of the mixture.

    [**Ans.** (*i*) 0.0385 bar, (*ii*) 64.5%, (*iii*) 0.025 kg/kg dry air, (*iv*) 0.63, (*v*) 28°C, (*vi*) 99.2 kJ]

12. The pressure and temperature of the air in a room is 1 bar and 28°C. If the relative humidity is found to be 30 per cent, determine :

    (*i*) The partial pressure of the water vapour and dew point,

    (*ii*) The specific volume of each constituent, and

    (*iii*) The specific humidity.

    [**Ans.** (*i*) 0.0378 bar, 8.8°C ; (*ii*) $v_{air} = 0.874\ m^3/kg$, $v_{vap} = 122.7\ m^3/kg$; (*iii*) 0.00712 kg/kg dry air]

13. 100 m^3 of air per minute at 35°C DBT and 60% relative humidity is cooled to 20°C DBT by passing through a cooling coil. Find the following :

    (*i*) Capacity of cooling coil in kJ/h

    (*ii*) Amount of water vapour removed per hour, and

    (*iii*) Relative humidity of air coming out and its wet-bulb temperature.

    [**Ans.** (*i*) 1037088 kJ/h, (*ii*) 465.36 kg/h, (*iii*) 100%, 20°C]

14. Atmospheric air at 38°C and 40 per cent relative humidity is to be cooled and dehumidified to a state of saturated air at 10°C. The mass rate of flow of atmospheric air entering the dehumidifier is 45.4 kg/h. Neglecting any pressure drop, determine :

    (*i*) The mass of water removed ; (*ii*) The quantity of heat removed.

    [**Ans.** (*i*) 0.397 kg/h, (*ii*) 2287 kJ/h]

15. 1 kg of air at 24°C and a relative humidity of 70% is to be mixed adiabatically in a steady state, steady flow device with 1 kg of air at 16°C and a relative humidity of 10%. Assuming that the mixing is to be carried out at a constant pressure of 1.0 atm, determine the temperature and relative humidity of the stream leaving the device. [**Ans.** 19.5°C, 50%]

16. An air-water vapour mixture enters an adiabatic saturator at 30°C and leaves at 20°C, which is the adiabatic saturation temperature. The pressure remains constant at 1 bar. Determine the relative humidity and the humidity ratio of the inlet mixture. [**Ans.** 39.8%, 0.0107 kg/kg dry air]

11

Fuels and Combustion

(Including Chemical Thermodynamics)

11.1. INTRODUCTION

- Fuel may be *chemical* or *nuclear*. Here we shall consider briefly *chemical fuels* only.

A chemical fuel is a substance which releases heat energy on combustion. The principal combustible elements of each fuel are *carbon* and *hydrogen*. Though *sulphur* is a combustible element too but its presence in the fuel is considered to be *undesirable*.

- In **chemical thermodynamics** the study of systems involving chemical reactions is an important topic. *A chemical reaction may be defined as the rearrangement of atoms due to redistribution of electrons.* In a chemical reaction the terms, **reactants** and the **products** are frequently used. *'Reactants'* comprise of initial constituents which start the reaction while *'products'* comprise of *final constituents* which are formed by the chemical reaction. Although the basic principles which will be discussed in this chapter apply to any chemical reaction, here main attention will be focused on an important type of chemical reaction—**combustion**.

11.2. CLASSIFICATION OF FUELS

Fuels can be classified according to whether :

1. They occur in nature called **primary fuels** or are prepared called **secondary fuels** ;

2. They are in solid, liquid or gaseous state. The detailed classification of fuels can be given in a summary form as follows :

Type of fuel	*Natural (Primary)*	*Prepared (Secondary)*
Solid	Wood	Coke
	Peat	Charcoal
	Lignite coal	Briquettes

Liquid	Petroleum	Gasoline Kerosene Fuel oil Alcohol Benzol Shale oil
Gaseous	Natural gas	Petroleum gas Producer gas Coal gas Coke-oven gas Blast furnace gas Carburetted gas Sewer gas

11.3. SOLID FUELS

Coal. Its main constituents are carbon, hydrogen, oxygen, nitrogen, sulphur, moisture and ash. Coal passes through different stages during its formation from vegetation. These stages are enumerated and discussed below :

Plant debris—Peat—Lignite—Brown coal—sub-bituminous coal—Bituminous coal—Semi-bituminous coal—Semi-anthracite coal—Anthracite coal—Graphite.

Peat. It is the first stage in the formation of coal from wood. It contains huge amount of moisture and therefore it is dried for about 1 to 2 months before it is put to use. It is used as a domestic fuel in Europe and for power generation in Russia. In India it does not come in the categories of good fuels.

Lignites and brown coals. These are intermediate stages between peat and coal. They have a woody or often a clay like appearance associated with high moisture, high ash and low heat contents. Lignites are usually amorphous in character and impose transport difficulties as they break easily. They burn with a smoky flame. Some of this type are suitable for local use only.

Bituminous coal. It burns with long yellow and smoky flames and has high percentages of volatile matter. The average calorific value of bituminous coal is about 31350 kJ/kg. It may be of two types, namely *caking* or *noncaking.*

Semi-bituminous coal. It is softer than the anthracite. It burns with a very small amount of smoke. It contains 15 to 20 per cent volatile matter and has a tendency to break into small sizes during storage or transportation.

Semi-anthracite. It has less fixed carbon and less lustre as compared to true anthracite and gives out longer and more luminous flames when burnt.

Anthracite. It is very hard coal and has a shining black lustre. It ignites slowly unless the furnace temperature is high. It is non-caking and has high percentage of fixed carbon. It burns either with very short blue flames or without flames. The calorific value of this fuel is high to the tune of 35500 kJ/kg and as such is *very suitable for steam generation.*

Wood charcoal. It is obtained by destructive distillation of wood. During the process the volatile matter and water are expelled. The physical properties of the residue (charcoal), however depends upon the rate of heating and temperature.

Coke. It consists of carbon, mineral matter with about 2% sulphur and small quantities of hydrogen, nitrogen and phosphorus. It is solid residue left after the destructive distillation of certain kinds of coals. It is smokeless and clear fuel and can be produced by several processes. It is *mainly used in blast furnace* to produce heat and at the same time to reduce the iron ore.

Briquettes. These are prepared from fine coal or coke by compressing the material under high pressure.

11.4. LIQUID FUELS

The chief source of liquid fuels is *petroleum* which is obtained from wells under the earth's crust. These fuels have proved *more advantageous in comparison to sold fuels* in the following respects.

Advantages :

1. Require less space for storage.
2. Higher calorific value.
3. Easy control of consumption.
4. Staff economy.
5. Absence of danger from spontaneous combustion.
6. Easy handling and transportation.
7. Cleanliness.
8. No ash problem.
9. Non-deterioration of the oil in storage.

Petroleum. There are different opinions regarding the origin of petroleum. However, now it is accepted that petroleum has originated probably from organic matter like fish and plant life etc., by bacterial action or by their distillation under pressure and heat. It consists of a mixture of gases, liquids and solid hydrocarbons with small amounts of nitrogen and sulphur compounds. In India, the main sources of Petroleum are Assam and Gujarat.

Heavy fuel oil or crude oil is imported and then refined at different refineries. The refining of crude oil supplies the most important product called *petrol*. Petrol can also be made by polymerization of refinery gases.

Other liquid fuels are kerosene, fuels oils, colloidal fuels and alcohol.

11.5. GASEOUS FUELS

Natural gas. The main constituents of natural gas are *methane* (CH_4) and *ethane* (C_2H_6). It has calorific value nearly 21000 kJ/m^3. Natural gas is used alternately or simultaneously with oil for internal combustion engines.

Coal gas. Mainly consists of *hydrogen, carbon monoxide* and *hydrocarbons*. It is prepared by carbonisation of coal. It finds its use in boilers and sometimes used for commercial purposes.

Coke-oven gas. It is obtained during the production of coke by heating the bituminous coal. The volatile content of coal is driven off by heating and major portion of this gas is utilised in heating the ovens. This gas *must be thoroughly filtered before using in gas engines.*

Blast furnance gas. It is obtained from smelting operation in which air is forced through layers of coke and iron ore, the example being that of pig iron manufacture where this gas is produced as by product and contains about 20% carbon monoxide (CO). After filtering it may be blended with richer gas or used in gas engines directly. The heating value of this gas is very low.

Producer gas. It results from the partial oxidation of coal, coke or peat when they are burnt with an insufficient quantity of air. It is produced in specially designed retorts. It has low heating value and in general is suitable for large installations. It is also used in steel industry for firing open hearth furnaces.

Water or illuminating gas. It is produced by blowing steam into white hot coke or coal. The decomposition of steam takes place liberating free hydrogen, and oxygen in the steam combines with carbon to form carbon monoxide according to the reaction :

$$C + H_2O \longrightarrow CO + H_2$$

The gas composition varies as the hydrogen content if the coal is used.

Sewer gas. It is obtained from sewage disposal vats in which fermentation and decay occur. It consists of mainly marsh gas (CH_4) and is collected at large disposal plants. It works as a fuel for gas engines which in turn drive the plant pumps and agitators.

Gaseous fuels are becoming popular because of following *advantages* they possess.

Advantages :

1. Better control of combustion.
2. Much less excess air is needed for complete combustion.
3. Economy in fuel and more efficiency of furnace operation.
4. Easy maintenance of oxidizing or reducing atmosphere.
5. Cleanliness.
6. No problem of storage if the supply is available from public supply line.
7. The distribution of gaseous fuels even over a wide area is easy through the pipe lines and as such handling of the fuel is altogether eliminated.
8. Gaseous fuels give economy of heat and produce higher temperatures (as they can be preheated in regenerative furnances and thus heat from hot flue gases can be recovered).

11.6. BASIC CHEMISTRY

Before considering combustion problems it is necessary to understand the construction and use of chemical formulae. This involves elementary concepts which are discussed below briefly.

Atoms. It is not possible to divide the chemical elements *indefinitely*, and the *smallest particle which can take part in a chemical change* is called an **'atom'**. If an atom is split as in nuclear reaction, the *divided atom does not retain the original chemical properties*.

Molecules. It is rare to find elements to exist naturally as single atom. Some elements have atoms which exist in pairs, each pair forming a molecule (*e.g.* oxygen), and the atoms of each molecule are held together by stronger *inter-atomic forces*. The isolation of a molecule of oxygen would be tedious, but possible ; the isolation of an atom of oxygen would be a different prospect. The molecules of some substances are formed by the mating up of atoms of different elements. For example, water has a molecule which consists of two atoms of hydrogen and one atom of oxygen. The atoms of different elements have different masses and these values are important when a quantitative analysis is required. The actual masses are infinitesimally small, and the ratios of the masses of atoms are used. These ratios are indicated by **atomic weight** quoted on a scale which defines the atomic weight of oxygen as 16.

The symbols and molecular weights of some important elements, compounds and gases are given in Table 11.1.

Table 11.1. Symbols and Molecular weights

Elements / Compounds / Gases	*Molecule*		*Atom*	
	Symbol	*Molecular weight*	*Symbol*	*Molecular weight*
Hydrogen	H_2	2	H	1
Oxygen	O_2	32	O	16
Nitrogen	N_2	28	N	14
Carbon	C	12	C	12
Sulphur	S	32	S	32
Water	H_2O	18	—	—
Carbon monoxide	CO	28	—	—
Carbon dioxide	CO_2	44	—	—
Sulphur dioxide	SO_2	64	—	—
Marsh gas (Methane)	CH_4	16	—	—
Ethylene	C_2H_4	28	—	—
Ethane	C_2H_6	30	—	—

11.7. COMBUSTION EQUATIONS

- In a combustion chamber proportionate masses of air and fuel enter where the chemical reaction takes place, and then the combustion products pass to the exhaust. By the conservation of mass the mass flow remains constant (*i.e.*, total mass of *products* = total mass of *reactants*), but the reactants are chemically different from the products, and the products leave at a higher temperature. The *total number of atoms of each element concerned in the combustion remains constant, but the atoms are rearranged into groups having different chemical properties.* This information is expressed in the chemical equation which shows (*i*) the reactants and the products of combustion, (*ii*) the relative quantities of the reactants and products. The two sides of the equation must be **consistent**, each having the same number of atoms of each element involved.
- The oxygen supplied for combustion is *usually* provided by *atmospheric air*, and it is necessary to use accurate and consistent analysis of air by *mass* and by *volume*. It is usual in combustion calculations to take air as 23.3% O_2, 76.7% N_2 by mass, and 21% O_2, 79% N_2 by volume. The small traces of other gases in dry air are included in nitrogen, which is sometimes called *'atmospheric nitrogen'*.

Some important *combustion equations* are given below :

1. **Combustion of hydrogen**

$$2H_2 + O_2 = 2H_2O \qquad ...(11.1)$$

The above equation of combustion of hydrogen tell us that :

(*i*) Hydrogen reacts with water to form steam or water.

(*ii*) Two molecules of hydrogen react with one molecule of oxygen to give two molecules of steam or water,

i.e., $$2 \text{ volumes } H_2 + 1 \text{ volume } O_2 \longrightarrow 2 \text{ volumes } H_2O$$

The H_2O may be liquid or a vapour depending on whether the product has been cooled sufficiently to cause condensation.

The *proportions by mass* are obtained by using *atomic weights* as follows :

$$2H_2 + O_2 \longrightarrow 2H_2O$$

$$\therefore \qquad 2(2 \times 1) + 2 \times 16 \longrightarrow 2(2 \times 1 + 16)$$

i.e.,
$$4 \text{ kg } H_2 + 32 \text{ kg } O_2 \longrightarrow 36 \text{ kg } H_2O$$

or
$$\mathbf{1 \text{ kg } H_2 + 8 \text{ kg } O_2 \longrightarrow 9 \text{ kg } H_2O}$$

[The same proportions are obtained by writing the equation (11.1) as : $H_2 + \frac{1}{2} O_2 \longrightarrow H_2O$, and this is sometimes done.]

It will be noted from equation (11.1) that the total volume of the *reactants* is

$$2 \text{ volumes } H_2 + 1 \text{ volume } O_2 = 3 \text{ volumes.}$$

The total volume of the *product* is only 2 volumes. There is therefore a *volumetric contraction on combustion.*

Since the oxygen is accompanied by nitrogen if air is supplied for the combustion, then this nitrogen should be included in the equation. As nitrogen is *inert* as far as chemical reaction is concerned, it will appear on both sides of the equation.

With one mole of oxygen there are 79/21 moles of nitrogen, hence equation (11.1) becomes,

$$2H_2 + O_2 + \tfrac{79}{21} N_2 \longrightarrow 2H_2O + \tfrac{79}{21} N_2 \qquad \text{...(11.2)}$$

2. Combustion of carbon

(*i*) ***Complete combustion of carbon to carbon dioxide***

$$C + O_2 \longrightarrow CO_2 \qquad \text{...(11.3)}$$

and including the nitrogen,

$$C + O_2 + \tfrac{79}{21} N_2 \longrightarrow CO_2 + \tfrac{79}{21} N_2 \qquad \text{...(11.4)}$$

By volume :

$$0 \text{ volume C} + 1 \text{ volume } O_2 + \tfrac{79}{21} \text{ volumes } N_2 \longrightarrow 1 \text{ volume } CO_2 + \tfrac{79}{21} \text{ volumes } N_2 \qquad \text{...[11.4 (a)]}$$

The volume of carbon is written as zero since the volume of solid is negligible in comparison with that of a gas.

By mass :

$$12 \text{ kg C} + (2 \times 16) \text{ kg } O_2 + \tfrac{79}{21} (2 \times 14) \text{ kg } N_2 \longrightarrow (12 + 2 \times 16) \text{ kg } CO_2 + \tfrac{79}{21} (2 \times 14) N_2$$

i.e.,
$$12 \text{ kg C} + 32 \text{ kg } O_2 + 105.3 \text{ kg } N_2 \longrightarrow 44 \text{ kg } CO_2 + 105.3 \text{ kg } N_2$$

or
$$1 \text{ kg C} + \frac{8}{3} \text{ kg } O_2 + \frac{105.3}{12} \text{ kg } N_2 \longrightarrow \frac{11}{3} \text{ kg } CO_2 + \frac{105.3}{12} \text{ kg } N_2. \qquad \text{...[11.4 (b)]}$$

(*ii*) ***The incomplete combustion of carbon.*** The incomplete combustion of carbon occurs when there is an insufficient supply of oxygen to burn the carbon completely to carbon dioxide.

$$2C + O_2 \longrightarrow 2CO \qquad \text{...(11.5)}$$

and including the nitrogen,

$$2C + O_2 + \frac{79}{21} N_2 \longrightarrow 2CO + \frac{79}{21} N_2 \qquad \text{...(11.6)}$$

By mass :

$$(2 \times 12) \text{ kg C} + (2 \times 16) \text{ kg } O_2 + \frac{79}{21} (2 \times 14) \text{ kg } N_2 \longrightarrow 2(12 + 16) \text{ kg CO} + \frac{79}{21} (2 \times 14) \text{ kg } N_2$$

i.e.,
$$24 \text{ kg C} + 32 \text{ kg } O_2 + 105.3 \text{ kg } N_2 \longrightarrow 56 \text{ kg CO} + 105.3 \text{ kg } N_2$$

or
$$1 \text{ kg C} + \frac{4}{3} \text{ kg } O_2 + \frac{105.3}{24} \text{ kg } N_2 \longrightarrow \frac{7}{3} \text{ kg CO} + \frac{105.3}{24} \text{ kg } N_2 \qquad \text{...[11.6 (a)]}$$

If a further supply of oxygen is available then the combustion can continue to completion,

$$2CO + O_2 + \tfrac{79}{21}\,N_2 \longrightarrow 2CO_2 + \tfrac{79}{21}\,N_2 \qquad \text{...(11.7)}$$

By mass :

$$56 \text{ kg } CO + 32 \text{ kg } O_2 + \frac{79 \times 28}{21} \text{ kg } N_2 \longrightarrow 88 \text{ kg } CO_2 + \frac{79 \times 28}{21} \text{ kg } N_2$$

or

$$1 \text{ kg } CO + \frac{4}{7} \text{ kg } O_2 + \frac{105.3}{56} \text{ kg } N_2 \longrightarrow \frac{11}{7} \text{ kg } CO_2 + \frac{105.3}{56} \text{ kg } N_2. \qquad \text{...[11.7 (a)]}$$

11.8. THEORETICAL AIR AND EXCESS AIR

The *minimum amount of air* that supplies sufficient oxygen for the complete combustion of all the carbon, hydrogen, and any other elements in the fuel that may oxidise is called the **"theoretical air"**. When complete combustion is achieved with theoretical air, the products contain no oxygen.

In practice, it is found that complete combustion is not likely to be achieved unless the amount of air supplied is somewhat greater than the theoretical amount. Thus 150 per cent theoretical air means that air actually supplied is 1.5 times the theoretical air.

The complete combustion of methane with minimum amount of theoretical air and 150 per cent theoretical air respectively is written as :

$$CH_4 + 2O_2 + 2\left(\frac{79}{21}\right) N_2 \longrightarrow CO_2 + 2H_2O + 2\left(\frac{79}{21}\right) N_2 \qquad \text{...(11.8)}$$

$$CH_4 + 2(1.5)\,O_2 + 2\left(\frac{79}{21}\right)(1.5)\,N_2 \longrightarrow CO_2 + 2H_2O + O_2 + 3\left(\frac{29}{21}\right) N_2 \qquad \text{...(11.9)}$$

(with 150 per cent theoretical air)

The amount of air actually supplied may also be expressed in terms of per cent excess air. The excess air is the amount of air supplied over and above the theoretical air. Thus 150 per cent theoretical air is equivalent to 50 per cent excess air.

Note. For complete combustion of fuel we need air. As per theoretical basis there is a minimum amount of air which is required by the fuel to burn completely, *but always, air in excess is used because whole of air supplied for combustion purposes does not come in contact with the fuel completely and as such portion of fuel may be left unburnt*. But if a large quantity of excess air is used it exercises a *cooling effect on combustion process which however can be avoided by preheating the air. The weight of excess air supplied can be determined from the weight of oxygen which is left unused*. The amount of excess air supplied varies with the type of fuel and the firing conditions. It may approach a value of 100% but modern practice is to use 25% to 50% excess air.

11.9. STOICHIOMETRIC AIR FUEL (A/F) RATIO

Stoichiometric (or chemically correct) *mixture* of air and fuel is one that contains *just sufficient oxygen for complete combustion of the fuel*.

A *weak mixture* is one which has an *excess of air*.

A *rich mixture* is one which has a *deficiency* of air.

The percentage of excess air is given as under :

$$\%\text{age excess air} = \frac{\text{Actural A/F ratio} - \text{Stoichiometric A/F ratio}}{\text{Stoichiometric A/F ratio}} \qquad \text{...(11.10)}$$

(where A and F denote *air* and *fuel* respectively)

The ratios are expressed as follows :

For *gaseous fuels*	*By volume*
For solid and liquid fuels	*By mass*

For *boiler* plant the mixture is usually greater than 20% weak ; for *gas turbines* it can be as much as 300% weak. *Petrol engines* have to meet various conditions of load and speed, and operate over a wide range of mixture strength. The following definition is used :

$$\text{Mixture strength} = \frac{\text{Stoichiometric A/F ratio}}{\text{Actual A/F ratio}} \qquad \text{...(11.11)}$$

The working value range between 80% (weak) and 120% (rich).

Note. The reciprocal of the air fuel ratio is called the *fuel-air (F/A) ratio*.

11.10. AIR-FUEL RATIO FROM ANALYSIS OF PRODUCTS

When analysis of combustion products is known air-fuel ratio can be calculated by the following methods :

1. **Fuel composition known**

(*i*) Carbon balance method (*ii*) Hydrogen balance method

(*iii*) Carbon-hydrogen balance method.

2. **Fuel composition unknown**

(*i*) Carbon-hydrogen balance method.

1. **Fuel composition known**

(*i*) *Carbon balance method.* When the fuel composition is known, the carbon balance method is quite accurate if *combustion takes place with excess air and when free (solid) carbon is not present in the products*. It may be noted that the Orsat analysis will not determine the quantity of solid carbon in the products.

(*ii*) *Hydrogen balance method.* This method is used when solid carbon is suspected to be present.

(*iii*) *Carbon-hydrogen balance method.* This method may be employed *when there is some uncertainty about the nitrogen percentage reported by the Orsat analysis.*

2. **Fuel composition unknown**

When the fuel composition is not known the carbon-hydrogen balance method has to be employed.

11.11. HOW TO CONVERT VOLUMETRIC ANALYSIS TO WEIGHT ANALYSIS ?

The conversion of volumetric analysis to weight analysis involves the following steps :

1. Multiply the volume of each constituent by its molecular weight.
2. Add all these weights and then divide each weight by the total of all and express it as percentage.

11.12. HOW TO CONVERT WEIGHT ANALYSIS TO VOLUMETRIC ANALYSIS ?

1. Divide the weight of each constituent by its molecular weight.
2. Add up these volumes and divide each volume by the total of all and express it as a percentage.

11.13. WEIGHT OF CARBON IN FLUE GASES

The weight of carbon contained in one kg of flue or exhaust gas can be calculated from the amounts of CO_2 and CO contained in it.

In eqn. [11.4 (*b*)], it was shown that 1 kg of carbon produces 11/3 kg of CO_2 when completely burnt. Hence 1 kg of CO_2 will contain 3/11 kg of carbon.

In eqn. [11.6 (*a*)], it can be seen that 1 kg of carbon produces 7/3 kg of CO, hence 1 kg CO contains 3/7 kg of carbon.

Therefore, weight of carbon per kg of fuel

$$= \left(\frac{3}{11}CO_2 + \frac{3}{7}CO\right)$$

where CO_2 and CO are the quantities of carbon dioxide and carbon monoxide present in 1 kg of flue or exhaust gas.

11.14. WEIGHT OF FLUE GASES PER KG OF FUEL BURNT

Due to supply of air, the weight of flue gas or exhaust gas is always more than that of fuel burnt. The actual weight of dry flue gases can be obtained by comparing the weight of carbon present in the flue gases with the weight of carbon in the fuel, since there is no loss of carbon during the combustion process. As the analysis of the exhaust gases is volumetric, so this *must first be reduced to weight analysis.*

Also, *total weight of carbon in one kg of flue gas is*

$$= \left(\frac{3}{11}CO_2 + \frac{3}{7}CO\right)$$

∴ The *weight of flue gas/kg of fuel burnt*

$$= \frac{\text{Weight of carbon in one kg of fuel}}{\text{Weight of carbon in one kg of flue gas}}.$$

11.15. ANALYSIS OF EXHAUST AND FLUE GAS

The combustion products are mainly gaseous. When a sample is taken for analysis it is usually cooled down to a temperature which is below the saturation temperature of the steam present. The *steam content* is therefore *not included* in the analysis, which is then quoted as the *analysis of the dry products*. Since the products are gaseous, it is usual to quote the analysis by *volume. An analysis which includes the steam in the exhaust is called a wet analysis.*

Practical analysis of combustion products :

The most common means of analysis of the combustion products is the ***Orsat apparatus*** which is described below :

Construction. An Orsat's apparatus consists of the following :

(*i*) A burette

(*ii*) A gas cleaner

(*iii*) Four absorption pipettes 1, 2, 3, 4.

The pipettes are interconnected by means of a manifold fitted with cocks S_1, S_2, S_3 and S_4 and contain different chemicals to absorb carbon dioxide (CO_2), carbonmonoxide (CO) and oxygen (O_2). Each pipette is also fitted with a number of small glass tubes which provide a greater amount of surface. These tubes are wetted by the absorbing agents and are exposed to the gas under analysis. The measuring burrette is surrounded by a *water jacket* to prevent, changes in temperature and density of the gas. The pipettes 1, 2, 3, 4 contain the following chemicals :

Pipette 1 : Contains '*KOH*' (caustic soda) *to absorb* CO_2 (carbon dioxide)

Pipette 2 : Contains *an alkaline solution of 'pyrogallic acid' to absorb* O_2 (oxygen)

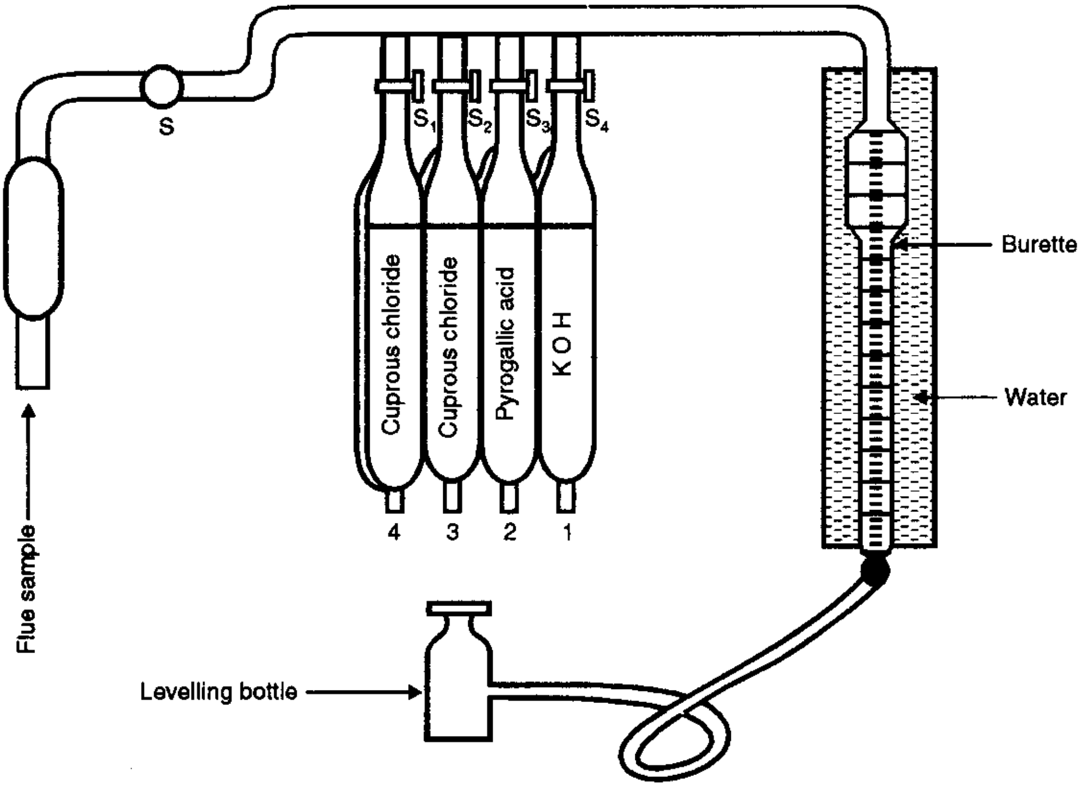


Fig. 11.1. Orsat's apparatus.

Pipette 3, 4 : Contain an *acid solution of 'cuprous chloride' to absorb CO* (carbonmonoxide)

Furthermore the apparatus has a *levelling bottle* and a three way cock to connect the apparatus either to gases or to atmosphere.

Procedure. 100 cm^3 of gas whose analysis is to be made is drawn into the bottle by lowering the levelling bottle. The stop cock S_4 is then opened and the whole flue gas is forced to pipette 1. The gas remains in this pipette for sometime and most of the carbondioxide is absorbed. The levelling bottle is then lowered to allow the chemical to come to its original level. The volume of gas thus absorbed is read on the scale of the measuring bottle. The flue gas is then forced through the pipette 1 for a number of times to ensure that the whole of the CO_2 is absorbed. Further, the remaining flue gas is then forced to the pipette 2 which contains pyrogallic acid to absorb whole of 0_2. The reading on the measuring burette will be the sum of volume of CO_2 and 0_2. The oxygen content can then be found out by subtraction. Finally, as before, the sample of gas is forced through the pipettes 3 and 4 to absorb carbonmonoxide completely.

The amount of nitrogen in the sample can be determined by subtracting from total volume of gas the sum of CO_2, CO and 0_2 contents.

Orsat apparatus *gives an analysis of the dry products of combustion*. Steps may have been taken to remove the steam from the sample by condensing, but as the sample is collected over water it becomes saturated with water. The resulting analysis is nevertheless a true analysis of the dry products. This is because the volume readings are taken at a constant temperature and pressure, and the partial pressure of the vapour is constant. This means that the sum of the partial pressures of the remaining constituents is constant. The vapour then occupies the same proportion of the total volume at each measurement. Hence the vapour does not affect the result of the analysis.

Note. Quantitatively the dry product analysis can be used to calculate A/F ratio. This method of obtaining the A/F ratio is not so reliable as direct measurement of air consumption and fuel consumption of the engine. More caution is required when analysing the products of consumption of a *solid fuel* since some of the products do not appear in the flue gases (*e.g.* ash and unburnt carbon). The residual solid must be analysed as well in order to determine the carbon content, if any. With an engine using *petrol or diesel fuel* the exhaust may include unburnt particles of carbon and this quantity will not appear in the analysis. The exhaust from internal combustion engines may contain also some CH_4 and H_2 due to incomplete combustion. Another piece of equipment called the **Heldane apparatus** measures the CH_4 content as well as CO_2, O_2 and CO.

11.16. INTERNAL ENERGY AND ENTHALPY OF FORMATION

The first law of thermodynamics can be applied to any system. Non-flow and steady-flow energy equations deducted from this law must be applicable to systems undergoing combustion processes.

It has been proved experimentally that the energy released, when a unit mass of a fuel undergoes complete combustion, depends on the *temperature at which the process is carried out.* Thus such quantities quoted are related to temperature. Now it will be shown that if the energy released by a fuel at one temperature is known then it can be calculated at other temperatures.

The process of combustion is defined as taking place from reactants at a state identified by the reference temperature T_0 and another property, either pressure or volume, to products at the same state.

Let U_{R_0} = Internal energy of the *reactants* (which is a mixture of fuel and air) at T_0,

U_{P_0} = Internal energy of *products* of combustion at T_0,

U_{R_1} = Internal energy of *reactants* at temperature T_1,

U_{P_1} = Internal energy of *products* at temperature T_1,

U_{R_2} = Internal energy of *reactants* at temperature T_2,

U_{P_2} = Internal energy of *products* at temperature T_2,

ΔU_0 = Constant volume heat of combustion,

Q = Heat transferred to the surroundings during the process, and

W = Work obtained during combustion process.

Analysis for a non-flow process involving combustion at 'constant volume' :

When the combustion process is carried out at *constant volume* then the non-flow energy equation, $Q = (U_2 - U_1) + W$, can be applied to give

$$Q = (U_{P_0} - U_{R_0}) \qquad ...(11.12)$$

where, $W = 0$ for constant volume combustion,

$U_1 = U_{R_0}$, and

$U_2 = U_{P_0}$.

The internal energy change is independent of the path between the two states and depends only on the initial and final values and is given by the quantity Q. This is illustrated in Fig. 11.2 and *property diagram* of Fig. 11.3. The heat so transferred is called the *internal energy of combustion* at T_0 (or *constant volume heat of combustion*), and is denoted by ΔU_0. Thus,

$$\Delta U_0 = U_{P_0} - U_{R_0} \qquad ...(11.13)$$

ΔU_0 is a *negative quantity* since the internal energy of the reactants includes the potential chemical energy and heat is transferred *from* the system.

It may be noted that in case of real constant volume combustion processes the initial and final temperatures will not be same as T_0 (reference temperature). The change in internal energy, for analytical purposes, between reactants at state 1 to products at state 2 can be considered in the following *three steps* (stages) :

(*i*) The change for the reactants from state 1 to T_0.

(*ii*) The constant volume combustion process from reactants to products at T_0.

(*iii*) The change for the products from T_0 to state 2.

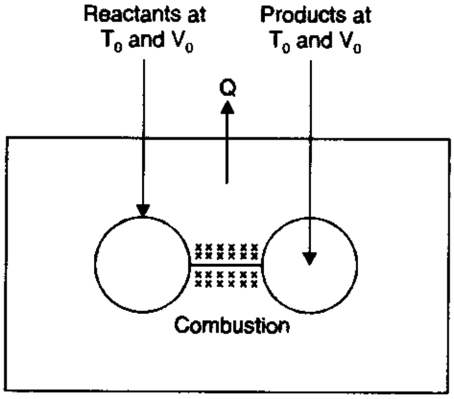


Fig. 11.2

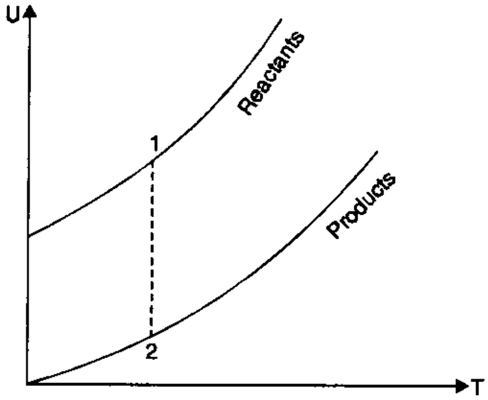


Fig. 11.3

The entire process can be thought of as taking place in piston-cylinder device as shown in Fig. 11.4.

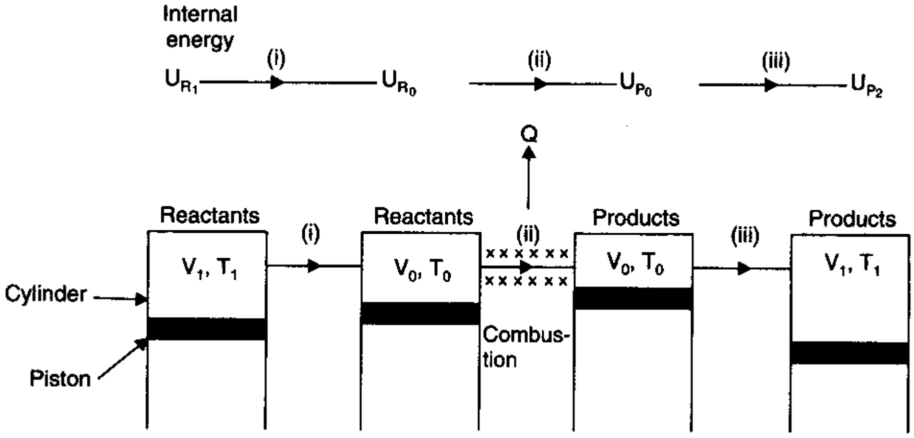


Fig. 11.4

Thus $U_2 - U_1$, the change in internal energy between states 1 and 2, can be written as $(U_{P_2} - U_{R_1})$ to show the chemical changes involved and this can be further expanded for analytical purposes as follows :

$$U_{P_2} - U_{R_1} = (U_{P_2} - U_{P_0}) + (U_{P_0} - U_{R_0}) + (U_{R_0} - U_{R_1})$$

i.e.,

$$U_{P_2} - U_{R_1} = \underbrace{(U_{P_2} - U_{P_0})}_{\substack{\text{Products} \\ (iii)}} + \underset{(ii)}{\Delta U_0} + \underbrace{(U_{R_0} - U_{R_1})}_{\substack{\text{Reactants} \\ (i)}} \quad \text{...(11.14)}$$

The values of $(U_{R_0} - U_{R_1})$ and $(U_{P_2} - U_{P_0})$ can be calculated from the following relations :

$$U_{R_0} - U_{R_1} = \sum_R n_i(u_{i_0} - u_{i_1}) \quad \text{...(11.15)}$$

where, u_i = Tabulated value of the internal energy for any constituent at the required temperature T_0 or T_1 in heat unit per mole

n_i = Number of moles of the constituent, and

$\sum\limits_R$ = Summation for all the constituents of the reactants denoted by i.

If *mass* base is used for tabulated values or calculation, then

$$U_{R_0} - U_{R_1} = \sum_R m_i(u_{i_0} - u_{i_1}) \quad \text{...(11.16)}$$

where, u_i = Internal energy per unit mass.

The above expression in terms of the *specific heats* (average values for the required temperature range) may be written as

$$U_{R_0} - U_{R_1} = \sum_R m_i c_{vi}(T_0 - T_1) = (T_0 - T_1) \sum_R m_i c_{vi} \quad \text{...(11.17)}$$

For *products*, similar expressions may be written as :

$$U_{P_2} - U_{P_0} = \sum_P n_i(u_{i_2} - u_{i_0}) \quad \text{... } \textit{on mole basis}$$

$$U_{P_2} - U_{P_0} = \sum_P m_i(u_{i_2} - u_{i_0}) \quad \text{... } \textit{on mass basis}$$

$$U_{P_2} - U_{P_0} = \sum_P m_i c_{vi}(T_2 - T_0)$$

$$= (T_2 - T_0) \sum_P m_i c_{vi} \quad \text{... in terms of } \textit{mean specific heats}$$

It may be noted that $n_i C_{vi} = m_i c_{vi}$

Analysis for a steady flow or 'constant pressure' combustion process :

In such an analysis the *changes in enthalpy (H)* are important. An analysis carried out as above will give the following expressions :

$$H_{P_2} - H_{R_1} = \underbrace{(H_{P_2} - H_{P_0})}_{\text{Products}} + \Delta H_0 + \underbrace{(H_{R_0} - H_{R_1})}_{\text{Reactants}} \quad \text{...(11.18)}$$

where, $\Delta H_0 = H_{P_0} - H_{R_0}$, and is always *negative* ...(11.19)

[ΔH_0 = *Enthalpy of combustion* at T_0 or the *constant pressure heat of combustion* at T_0]

Expressions for change of enthalpy of reactants and products :

Reactants :

$$H_{R_0} - H_{R_1} = \sum_R n_i(h_{i_0} - h_{i_1}) \quad \text{... } \textit{on mole basis} \quad \text{...(11.20)}$$

$$H_{R_0} - H_{R_1} = \sum_R m_i(h_{i_0} - h_{i_1}) \quad \text{... on mass basis} \quad ...(11.21)$$

$$H_{R_0} - H_{R_1} = \sum_R m_i c_{pi}(T_0 - T_1)$$

$$= (T_0 - T_1) \sum_R m_i c_{pi} \quad \text{... in terms of mean specific heats}$$

Products : $$H_{P_2} - H_{P_0} = \sum_P n_i(h_{i_2} - h_{i_0}) \quad \text{... on mole basis} \quad ...(11.22)$$

$$H_{P_2} - H_{P_0} = \sum_P m_i(h_{i_2} - h_{i_0}) \quad \text{... on mass basis} \quad ...(11.23)$$

$$H_{P_2} - H_{P_0} = \sum_P m_i c_{pi}(T_2 - T_0)$$

$$= (T_2 - T_0) \sum_P m_i c_{pi} \quad \text{... in terms of mean specific heats} \quad ...[11.23\ (a)]$$

It may be noted that $n_i C_{pi} = m_i c_{pi}$

From the definition of the enthalpy of a perfect gas

$$H = U + pV = U + nR_0T$$

So if we are concerned only with gaseous mixtures in the reaction then for products and reactants

$$H_{P_0} = U_{P_0} + n_P R_0 T_0$$

and $$H_{R_0} = U_{R_0} + n_R R_0 T_0$$

where n_P and n_R are the moles of products and reactants respectively and the temperature is the reference temperature T_0.

Thus, using eqns. (11.13) and (11.19), we have

$$\Delta H_0 = \Delta U_0 + (n_P - n_R)R_0T_0 \quad ...(11.24)$$

If there is no change in the number of moles during the reaction or if the reference temperature is absolute zero, then ΔH_0 and ΔU_0 will be equal.

11.17. ENTHALPY OF FORMATION (ΔH_f)

A combustion reaction is a particular kind of chemical reaction in which products are formed from reactants with the release or absorption of energy as heat is transferred to as from the surroundings. In some substances like hydrocarbon fuels which are many in number and complex in structure the *heat of reaction* or *combustion* may be calculated on the basis of known values of the enthalpy of formation, ΔH_f of the constituent of the reactants and products at the temperature T_0 (reference temperature). *The enthalpy of formation (ΔH_f) is the increase in enthalpy when a compound is formed from its constituent elements in their natural form and in a standard state.* The standard state is 25°C, and 1 atm. pressure, but it must be borne in mind that not all substances can exist in natural form, *e.g.* H_2O cannot be a vapour at 1 atm. and 25°C.

The expression of a particular reaction, for calculation purposes, may be given as :

$$\Delta H_0 = \sum_P n_i \Delta H_{f_i} - \sum_R n_i \Delta H_{f_i} \quad ...(11.25)$$

Typical values of ΔH_f for different substances at 25°C (298 K) in **kJ/mole** are given below :

S. No.	*Substance*	*Formula*	*State*	ΔH_f
1.	Oxygen	O	gas	249143
		O_2	gas	zero
2.	Water	H_2O	liquid	-285765
			vapour	-241783
3.	Carbon	C	gas	714852
			diamond	1900
			graphite	zero
4.	Carbon monoxide	CO	gas	-111508
5.	Carbon dioxide	CO_2	gas	-393443
6.	Methane	CH_4	gas	-74855
7.	Methyl alcohol	CH_3OH	vapour	-240532
8.	Ethyl alcohol	C_2H_5OH	vapour	-281102
9.	Ethane	C_2H_6	gas	-83870
10.	Ethene	C_2H_4	gas	51780
11.	Propane	C_3H_8	gas	-102900
12.	Butane	C_4H_{10}	gas	-125000
13.	Octane	C_8H_{18}	liquid	-247600

11.18. CALORIFIC OR HEATING VALUES OF FUELS

The "calorific value or heating value" of the fuel is defined as the energy liberated by the complete oxidation of a unit mass or volume of a fuel. It is expressed in kJ/kg for solid and liquid fuels and kJ/m^3 for gases.

If a fuel contains hydrogen water will be formed as one of the products of combustion. If this *water is condensed, a large amount of heat will be released* than if the water exists in the vapour phase. For this reason *two heating values* are defined ; the *higher or gross heating value* and the *lower* or *net heating value.*

The *higher heating value,* HHV, is obtained when the *water formed by combustion is completely condensed.*

The *lower heating value,* LHV, is obtained when the water formed by combustion exists *completely in the vapour phase.*

Thus :

$$(\text{HHV})_p = (\text{LHV})_p + m\, h_{fg} \quad \text{...(11.26)}$$

$$(\text{HHV})_v = (\text{LHV})_v + m(u_g - u_f) \quad \text{...(11.27)}$$

where
m = Mass of water formed by combustion,
h_{fg} = Enthalpy of vaporisation of water, kJ/kg,
u_g = Specific internal energy of vapour, kJ/kg, and
u_f = Specific internal energy of liquid, kJ/kg.

In almost all practical cases, the water vapour in the products is vapour, the lower value is the one *which usually applies.*

11.19. DETERMINATION OF CALORIFIC OR HEATING VALUES

The calorific value of fuels can be determined either from chemical analysis or in the laboratory.

11.19.1. Solid and Liquid Fuels

Dulong's formula. Dulong suggested a formula for the calculation of the calorific value of the solid or liquid fuels from their chemical composition which is as given below.

Gross calorific value

or $$\text{H.H.V.} = \frac{1}{100}\left[33800\,\text{C} + 144000\left(\text{H} - \frac{\text{O}}{8}\right) + 9270\,\text{S}\right] \text{kJ/kg} \qquad \ldots(11.28)$$

where C, H, O and S are carbon, hydrogen, oxygen and sulphur in percentages respectively in 100 kg of fuel. In the above formula the oxygen is assumed to be in combination with hydrogen and only extra surplus hydrogen supplies the necessary heat.

Laboratory method (Bomb calorimeter)

The calorific value of solid and liquid fuels is determined in the laboratory by '*Bomb calorimeter*'. It is so named because its shape resembles that of a bomb. Fig. 11.5 shows the schematic sketch of a bomb calorimeter.

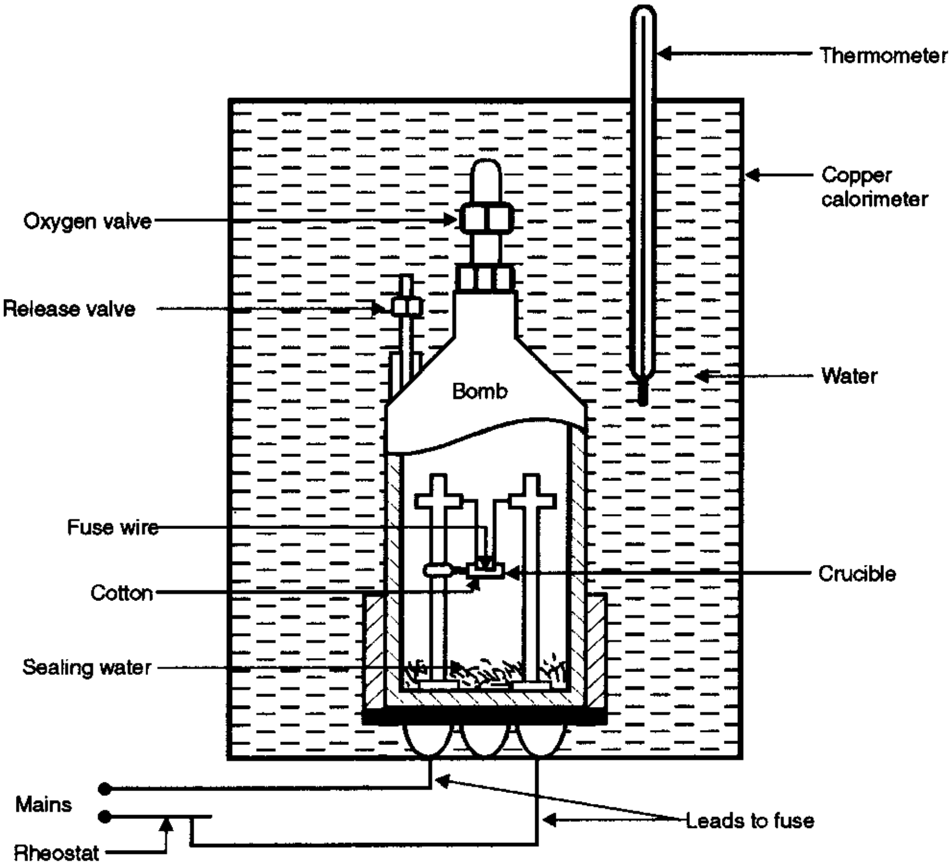


Fig. 11.5. Bomb calorimeter.

The *calorimeter* is made of austenitic steel which provides considerable resistance to corrosion and enables it to withstand high pressure. In the calorimeter is a *strong cylindrical bomb* in which combustion occurs. The bomb has *two valves* at the top. One supplies oxygen to the bomb and other releases the exhaust gases. A *crucible* in which a weighted quantity of fuel sample is burnt is arranged between the *two electrodes* as shown in Fig. 11.5. The calorimeter is fitted with *water jacket* which surrounds the bomb. To reduce the losses due to radiation, calorimeter is further provided with a jacket of water and air. A stirrer for keeping the temperature of water uniform and a thermometer to measure the temperature up to an accuracy of 0.001°C are fitted through the lid of the calorimeter.

Procedure. To start with, about 1 gm of fuel sample is accurately weighed into the crucible and a fuse wire (whose weight is known) is stretched between the electrodes. It should be ensured that *wire is in close contact with the fuel.* To absorb the combustion products of sulphur and nitrogen 2 ml of water is poured in the bomb. Bomb is then supplied with pure oxygen through the valve to an amount of 25 atmosphere. The bomb is then placed in the weighed quantity of water, in the calorimeter. The stirring is started after making necessary electrical connections, and when the thermometer indicates a steady temperature fuel is fired and temperature readings are recorded after 1/2 minute intervals until maximum temperature is attained. The bomb is then removed ; the pressure slowly released through the exhaust valve and the contents of the bomb are carefully weighed for further analysis.

The heat released by the fuel on combustion is absorbed by the surrounding water and the calorimeter.

From the above data the calorific value of the fuel can be found in the following way :

Let w_f = Weight of fuel sample (kg),
w = Weight of water (kg),
C = Calorific value (higher) of the fuel (kJ/kg),
w_e = Water equivalent of calorimeter (kg),
t_1 = Initial temperature of water and calorimeter,
t_2 = Final temperature of water and calorimeter,
t_c = Radiation corrections, and
c = Specific heat of water.

Heat released by the fuel sample = $w_f \times C$

Heat received by water and calorimeter

$$= (w_w + w_e) \times c \times [(t_2 - t_1) + t_c].$$

Heat lost = Heat gained

$$\therefore \quad w_f \times C = (w + w_e) \times c \times [(t_2 - t_1) + t_c]$$

i.e.,

$$C = \frac{(w + w_e) \times c \times [(t_2 - t_1) + t_c]}{w_f} \qquad ...(11.29)$$

[Value of c is 4.18 in SI units and unity in MKS units.]

Note 1. Corrections pertain to the heat of oxidation of fuse wire, heat liberated as a result of formation of sulphuric and nitric acids in the bomb itself.

2. It should be noted that *bomb calorimeter measures the higher or gross calorific value because the fuel sample is burnt at a constant volume in the bomb.* Further the bomb calorimeter will measure the H.C.V. directly if the bomb contains adequate amount of water before firing to saturate the oxygen. Any water formed from combustion of hydrogen will, therefore, be condensed.

The procedure of determining calorific values of *liquid fuels* is similar to that described above. However, if the liquid fuel sample is volatile, it is weighed in a glass bulb and broken in a tray just before the bomb is closed. In this way the loss of volatile constituents of fuels during weighing operation is prevented.

Example 11.1. *The following particulars refer to an experimental determination of the calorific value of a sample of coal containing 88% C and 4.2% H_2. Weight of coal = 0.848 gm,* *weight of fuse wire 0.027 gm, of calorific value 6700 J/gm, weight of water in the calorimeter = 1950 gm, water equivalent of calorimeter = 380 gm, observed temperature rise = 3.06°C, cooling correction = + 0.017°C.*

Find the higher and lower calorific values of the coal.

Solution. Percentage of *carbon* in coal = 88%
Percentage of *hydrogen* in coal = 4.2%
Weight of coal, $w_f = 0.848$ gm
Weight of fuse wire, $w_{fw} = 0.027$ gm
Weight of water in the calorimeter, $w = 1950$ gm
Water equivalent of calorimeter, $w_e = 380$ gm
Observed temperature rise $(t_2 - t_1)$ = 3.06°C
Cooling correction, $t_c = + 0.017$°C
∴ Corrected temperature rise $= (t_2 - t_1) + t_c$
= 3.06 + 0.017 = 3.077°C
Calorific value of fuse wire = 6700 J/gm
Heat received by water $= (w + w_e) \times 4.18 \times [(t_2 - t_1) + t_c]$
$= (1950 + 380) \times 4.18 \times 3.077 = 29968$ J
Heat given out by fuse wire $= w_{fw} \times$ calorific value $= 0.027 \times 6700 = 180.9$ J
Heat produced due to combustion of fuel $= 29968 - 180.9 = 29787$ J
∴ *Higher calorific value of fuel, H.C.V.* $= \dfrac{29787}{0.848} = 35126$ J/gm = **35126 kJ/kg. (Ans.)**
Steam produced per kg of coal $= 9 \times 0.042 = 0.378$ kg
Lower calorific value of coal, L.C.V. = H.C.V. $- 2465 \times 0.378 = 35126 - 931.7$
= **34194.3 kJ/kg. (Ans.)**

11.19.2. Gaseous Fuels

The calorific value of gaseous fuels can be determined by ***Junker's gas calorimeter.***

Fig. 11.6 illustrates *Junker's gas calorimeter.* Its principle is some what similar to Bomb calorimeter ; in respect that heat evolved by burning the gas is taken away by the water. In its simplest construction it consists of a combustion chamber in which the gas is burnt (in a gas burner). A water jacket through which a set of tubes called flues pass surrounds this chamber. Thermometers are incorporated at different places (as shown in Fig. 11.6) to measure the temperatures.

Procedure. A metered quantity of gas whose calorific value is to be determined is supplied to the gas burner via a gas meter which records its volume and a gas pressure regulator which measures the pressure of the gas by means of a manometer. When the gas burns the hot products of combustion travel upwards in the chamber and then downwards through the flues and finally escape to the atmosphere through the outlet. The temperature of the escaping gas is recorded by the thermometer fitted at the exit and this temperature should be as close to room temperature as possible so that entire heat of combustion is absorbed by water. The cold water enters the calorimeter near the bottom and leaves near the top. Water which is formed by condensation of steam is collected in a pot.

The quantity of gas used during the experiment is accurately measured by the meter and temperature of ingoing and outgoing water are indicated by the thermometers. From the above data the calorific value of the gas can be calculated.

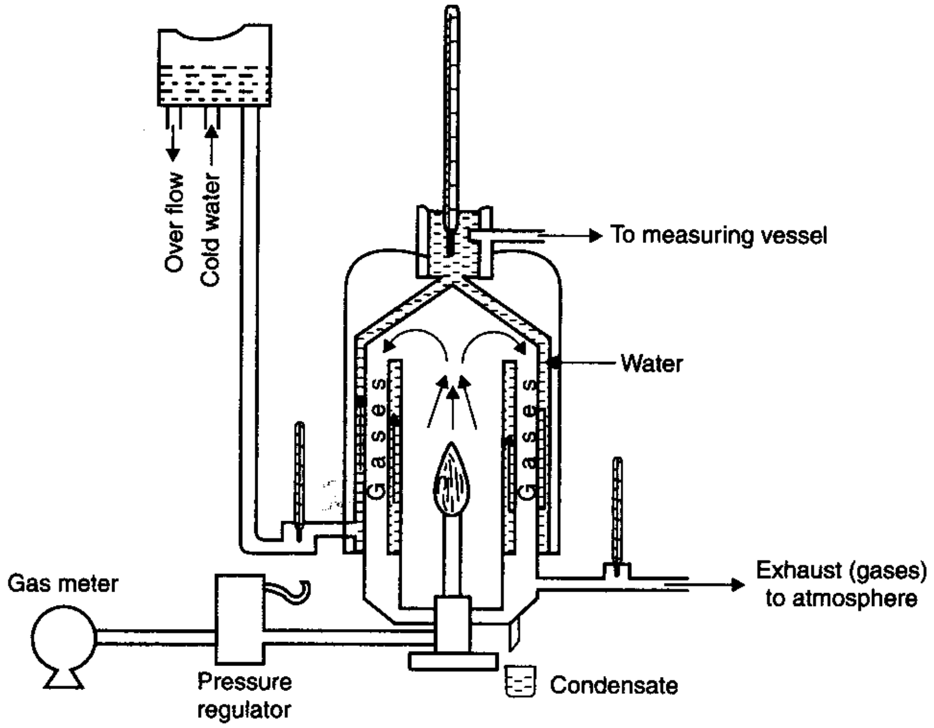


Fig. 11.6. Junker's gas calorimeter.

Note 1. The volume of gas used during the experiment should be converted to volume at S.T.P. (standard temperature and pressure *i.e.*, 15°C, 760 mm respectively).

2. The gross calorific value is obtained by dividing the heat given out by corrected volume of gas. The net or lower calorific value of the gas is obtained by subtracting from total heat the heat associated with condensed water (which is obtained by multiplying the weighed condensate by latent heat of vapourisation of water.).

Example 11.2. *Following results were obtained when a sample of gas was tested by Junker's gas calorimeter :*

Gas burnt in the calorimeter = 0.08 m³, Pressure of gas supply = 5.2 cm of water, Barometer = 75.5 cm of Hg. Temperature of gas = 13°C, Weight of water heated by gas = 28 kg, Temperature of water at inlet = 10°C, Temperature of water at outlet = 23.5°C, Steam condensed = 0.06 kg.

Determine the higher and lower calorific values per m³ of the gas at a temperature of 15°C and barometric pressure of 76 cm of Hg.

Solution. The volume of the gas is measured at a temperature of 13°C and pressure of 5.2 cm of water. Let us reduce this volume to S.T.P. by using the general gas equation,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$p_1 = 75.5 + \left(\frac{5.2}{13.6}\right) = 75.882 \text{ cm of Hg} \; ; \; T_1 = 273 + 13 = 286 \text{ K}$$

$$V_1 = 0.08 \text{ m}^3 \; ; \; p_2 = 76 \text{ cm of Hg}$$

$$V_2 = ? \; ; \; T_2 = 273 + 15 = 288 \text{ K}$$

$$\therefore \quad \frac{75.882 \times 0.08}{286} = \frac{76 \times V_2}{288}$$

i.e., $$V_2 = \frac{75.882 \times 0.08 \times 288}{286 \times 76} = 0.0804 \text{ m}^3$$

Heat received by water $= 28 \times 4.18 \times (23.5 - 10) = 1580$ kJ

Higher calorific value of fuel $= \frac{1580}{0.08} =$ **19750 kJ/m³. (Ans.)**

Amount of water vapour formed (*i.e.*, steam condensed) *per m³ of gas burnt* $= \frac{0.06}{0.08} = 0.75$ kg

Lower calorific value, L.C.V. = H.C.V. − 2465 × 0.75

= 19750 − 1848.7 = **17901.3 kJ/kg. (Ans.)**

11.20. ADIABATIC FLAME TEMPERATURE

In a given combustion process, that takes place *adiabatically* and with no work or changes in kinetic or potential energy involved, *the temperature of the products is referred to as the 'adiabatic flame temperature'.* With the assumptions of no work and no changes in kinetic or potential energy, this is the *maximum temperature* that can be achieved for the given reactants because any heat transfer from the reacting substances and any incomplete combustion would tend to lower the temperature of the products.

The following points are worthnoting :

(*i*) *The maximum temperature achieved through adiabatic complete combustion varies with the type of reaction and per cent of theoretical air supplied.*

An *increase in the air-fuel* ratio will effect a *decrease in the maximum temperature.*

(*ii*) For a given fuel and given pressure and temperature of the reactants, the *maximum adiabatic flame temperature that can be achieved is with a 'stoichiometric' mixture.*

(*iii*) The adiabatic flame temperature can be *controlled* by the *amount of excess air* that is used. This is important, for example, in *gas turbines*, where the maximum permissible temperature is determined by *metallurgical considerations* in the turbine, and *close control of the temperature of the products is essential.*

11.21. CHEMICAL EQUILIBRIUM

The calculation of the *adiabatic flame temperature* is based, in part, on the assumption that the reaction goes to *completion.* Owing to *dissociation,* complete conversion of the reactants to the products is not accomplished. As a consequence of the failure to achieve complete conversion of the reactants, the maximum reaction temperature cannot attain the level of the theoretical adiabatic flame temperature.

The combination of CO and O_2 produces CO_2 together with a release of energy. In an adiabatic system no heat is transferred to the surroundings, hence the temperature of the mixture of the products and reacting substances rises rapidly. As the mixture temperature increases to higher levels the rate of dissociation of the CO_2 becomes increasingly more pronounced. Since the dissociation of CO_2 requires absorption of energy, *a condition is reached where the rate of evolution and the rate of absorption of energy are in balance. At this point no further increase in temperature can be observed* and the reaction is in *chemical equilibrium.* For this condition

$$C + O_2 \rightleftharpoons CO_2$$

At each temperature of the equilibrium mixture the substances participating in the reaction exist in unique proportions. For the combustion of CO the right-hand side of the equation

$$CO + \tfrac{1}{2} O_2 = (1 - x)\, CO_2 + x\, CO + \tfrac{x}{2} O_2 \qquad \text{...(11.30)}$$

represents the distribution of the equilibrium products resulting from a reaction between CO and O_2. In this equation x denotes the fraction of dissociated CO_2. At low temperatures the fraction $(1 - x)$ approaches unity while at high temperatures $(1 - x)$ shows a substantial reduction in magnitude.

For the combustion of H_2 with O_2

$$H_2 + \frac{1}{2} O_2 \longrightarrow H_2O$$

and

$$H_2 + \frac{1}{2} O_2 = (1 - x)\, H_2O + x\, H_2 + \frac{x}{2} O_2 \qquad ...(11.31)$$

It is essential to distinguish between the effects of dissociation and the losses resulting from incomplete combustion of fuel. Incomplete combustion, which may be attributed to a number of factors, results in a discharge from the system of combustible substances. *Dissociation*, on the other hand, is of *transient nature*. Usually any appreciable degree of dissociation extends over a very short time interval at the highest level of temperature attained in the reaction. The gaseous products are likely to be discharged from the system at a temperature that is indicative of a low degree of dissociation. *For example, dissociation does not influence the heating value determined in a fuel calorimeter.* Although the maximum temperature attained in the calorimeter is limited by chemical equilibrium, the combustion process moves to completion with the decrease in the temperature of the products. The reduction in temperature is a result of heat transfer to the jacket water. Dissociation of the products is negligible at room temperature, which is essentially the calorimeter reaction temperature.

The temperature of the products discharged from the combustion chamber of the gas turbine power plant is limited to approximately 870°C by introduction of a *large quantity of excess air*. Absorption of energy in the water walls of a boiler furnace limits the outlet gas temperature to approximately 1100°C. The quantity of dissociated products at temperatures ranging upward to 1100°C is not appreciable. In the cylinder of I.C. engine, considerably higher maximum temperatures—that is, in excess of 1100°C are attained, hence in the analysis of this thermal power system consideration must be given to the effects of dissociation. Of particular significance is the effect of reduced maximum temperature on the system availability. As a result of heat transfer and work performed by the gaseous medium the products are discharged from the system at a temperature below the level at which an appreciable degree of dissociation is observed.

The proportions of the dissociated products in chemical equilibrium at temperature T are established from the *equilibrium constant*. The evaluation of the equilibrium constant is achieved in accordance with the analysis presented by Van't Hoff.

11.22. ACTUAL COMBUSTION ANALYSIS

In evaluating the performance of an actual combustion process a number of different parameters can be defined depending on the nature of the process and the system considered. The combustion efficiency in a gas turbine for instance can be defined as

$$\eta_{\text{combustion}} = \frac{(F/A)_{\text{ideal}}}{(F/A)_{\text{actual}}} \qquad ...(11.32)$$

where $(F/A)_{\text{ideal}}$ = Fuel-air ratio required for adiabatic and complete combustion and in which the products would attain the adiabatic flame temperature.

In case of a *steam generator (boiler)*

$$\eta_{\text{steam generator}} = \frac{\text{Heat transferred to steam / kg fuel}}{\text{Higher heating value of the fuel}} \qquad ...(11.33)$$

In case of an *internal combustion engine,*

$$\eta_{\text{thermal}} = \frac{W_{\text{actual}}}{\text{Heating value}} \quad \text{...(11.34)}$$

Example 11.3. *A coal sample gave the following analysis by weight, Carbon 85 per cent, Hydrogen 6 per cent, Oxygen 6 per cent, the remainder being incombustible. Determine minimum weight of air required per kg of coal for chemically correct composition.*

Solution.

Element, wt. (kg)	*O_2 required (kg)*
$C = 0.85$	$0.85 \times \frac{8}{3} = 2.27$
$H_2 = 0.06$	$0.06 \times 8 = 0.48$
$O_2 = 0.06$	— — — — — —
	Total $O_2 = 2.75$

Weight of O_2 to be supplied = Wt. of O_2 needed – Wt. of O_2 already present in fuel
= 2.75 – 0.06 = 2.69 kg

Weight of air needed $= 2.69 \times \frac{100}{23}$ = **11.70 kg. (Ans.)**

☞**Example 11.4.** *The percentage composition of sample of liquid fuel by weight is, C = 84.8 per cent, and H_2 = 15.2 per cent. Calculate (i) the weight of air needed for the combustion of 1 kg of fuel ; (ii) the volumetric composition of the products of combustion if 15 per cent excess air is supplied.*

Solution.

Element, wt. (kg)	*O_2 used (kg)*	*Dry products (kg)*
$C = 0.848$	$0.848 \times \frac{8}{3} = 2.261$	$\frac{0.848 \times 11}{3} = 3.109\ (CO_2)$
$H_2 = 0.152$	$0.152 \times 8 = 1.216$	
	Total $O_2 = 3.477$	

(*i*) **Minimum weight of air needed for combustion**

$= \frac{3.477 \times 100}{23}$ = **15.11 kg. (Ans.)**

Excess air supplied $= \frac{15.11 \times 15}{100} = 2.266$ kg

Wt. of oxygen in excess air $= \frac{2.266 \times 23}{100} = 0.521$ kg

Total air supplied for combustion = Minimum air + Excess air
= 15.11 + 2.266 = 17.376 kg

∴ Wt. of nitrogen (N_2) in flue gases $= \frac{17.376 \times 77}{100} = 13.38$ kg.

(*ii*) To get **volumetric composition of the product of combustion** let us use tabular method.

Name of gas	*Weight (x)*	*Molecular weight (y)*	*Proportional volume* $(z) = \frac{(x)}{(y)}$	*Percentage volume* $= \frac{(z)}{\Sigma(z)} \times 100$
CO_2	3.109	44	0.0707	12.51 per cent. **(Ans.)**
O_2	0.521	32	0.0163	2.89 per cent. **(Ans.)**
N_2	13.38	28	0.4780	84.60 per cent. **(Ans.)**
			$\Sigma z = 0.5650$	

☞**Example 11.5.** *Percentage volumetric analysis of a sample of flue gases of a coal fired boiler gave* $CO_2 = 10.4$; $CO = 0.2$; $O_2 = 7.8$ *and* $N_2 = 81.6$ *(by difference). Gravemetric percentage analysis of coal was* $C = 78$, $H_2 = 6$, $O_2 = 3$ *and incombustible* = 13. *Estimate :*

(*i*) *Weight of dry flue gases per kg of fuel.*

(*ii*) *Weight of excess air per kg of fuel.*

Solution.

Element, wt. (kg)	O_2 *reqd. (kg)*	*Dry products (kg)*
$C = 0.78$	$0.78 \times \frac{8}{3} = 2.08$	$0.78 \times \frac{11}{3} = 2.86\ (CO_2)$
$H_2 = 0.06$	$0.06 \times 8 = 0.48$	
$O_2 = 0.03$		
	Total $O_2 = 2.56$	

$$\text{Minimum wt. of air needed for combustion} = (2.56 - .03) \times \frac{100}{23} = 11 \text{ kg.}$$

(*i*) **Weight of dry flue gases per kg of fuel :**

To determine the wt. of flue gases per kg of fuel let us use tabular method to convert volumetric analysis to analysis by weight.

Name of gas	*Volume per* m^3 *of flue gas (x)*	*Molecular weight (y)*	*Relative volume* $z = x \times y$	*Weight per kg of flue gas* $= \frac{z}{\Sigma z}$
CO_2	0.104	44	4.576	0.1525
CO	0.002	28	0.056	0.0019
N_2	0.816	28	22.848	0.7616
O_2	0.078	32	2.496	0.0832
			$\Sigma z = 29.976$ (say 30)	

Amount of carbon present per kg of gases

= Amount of carbon in 0.1525 kg of CO_2 + Amount of carbon present in 0.0019 kg of CO

$$= \frac{3}{11} \times 0.1525 + \frac{3}{7} \times 0.0019 = 0.0416 + 0.0008 = 0.0424 \text{ kg.}$$

Also carbon in the fuel = 0.78 kg.

∴ *Weight of dry flue gas per kg of fuel*

$$= \frac{\text{Weight of carbon in 1 kg of fuel}}{\text{Weight of carbon in 1 kg of flue gas}} = \frac{0.78}{0.0424} = \mathbf{18.4\ kg.}\quad \textbf{(Ans.)}$$

(*ii*) **Weight of excess air per kg of fuel :**

Weight of excess oxygen per kg of flue gas $= 0.0832 - \frac{4}{7} \times .0019$

$$\left[\begin{array}{l} 2CO + O_2 = 2CO_2 \\ \ \ 56 \quad 32 \quad 88 \\ 1\text{ kg} + \frac{4}{7}\text{ kg} = \frac{11}{7}\text{ kg} \end{array}\right]$$

= 0.0832 – 0.0011 (allowing for unburnt carbon monoxide)

= 0.0821 kg.

Weight of excess O_2 per kg of fuel = 18.4 × 0.0821 = 1.51 kg

∴ *Weight of excess air per kg of fuel* $= \frac{1.51 \times 100}{23} =$ **6.56 kg. (Ans.)**

Example 11.6. *A single cylinder was supplied with a gas having the following percentage volumetric analysis ; CO = 5, CO_2 = 10, H_2 = 50, CH_4 = 25, N_2 = 10. The percentage volumetric analysis of dry gases was CO_2 = 8, O_2 = 6 and N_2 = 86. Determine the air-fuel ratio by volume.*

Solution. Combustion equations are :

$$2H_2 + O_2 = 2H_2O$$
$$1 \text{ vol} + 1/2 \text{ vol} = 1 \text{ vol}$$
$$2CO + O_2 = 2CO_2$$
$$1 \text{ vol} + 1/2 \text{ vol} = 1 \text{ vol}$$
$$CH_4 + 2O_2 = CO_2 + 2H_2O$$
$$1 \text{ vol} + 2 \text{ vol} = 1 \text{ vol} + 2 \text{ vol.}$$

Gas	*Vol (m^3)*	*O_2 needed (m^3)*	*Products (m^3)*	
			CO_2	*N_2*
CO	0.05	0.025	0.05	—
CO_2	0.10	—	0.10	—
H_2	0.50	0.25	—	—
CH_4	0.25	0.50	0.25	—
N_2	0.10	—	—	0.10
Total	1.0	0.775	0.4	0.10

Volume of air required $= 0.775 \times 100/21 = 3.69\ m^3$

Volume of nitrogen in the air $= 3.69 \times 79/100 = 2.92\ m^3$

Dry combustion products of 1 m^3 of gases (V) contain 0.4 m^3 of CO_2 + 0.1 m^3 of N_2 (as given in the table) + 2.92 m^3 of N_2 (from air supplied for complete combustion) = 3.42 m^3.

Excess air supplied $= \dfrac{O_2 \times V}{(21 - O_2)} = \dfrac{6.0 \times 3.42}{(21-6)} = \dfrac{20.52}{15} = 1.37 \text{ m}^3$

Total quantity of air supplied $= 3.69 + 1.37 = 5.06 \text{ m}^3$.

Air fuel ratio $= \dfrac{\text{Volume of air}}{\text{Volume of fuel}} = \dfrac{5.06}{1} = \mathbf{5}$ **(say). (Ans.)**

☞**Example 11.7.** *The following is the ultimate analysis of a sample of petrol by weight :*

Carbon = 85 per cent ; Hydrogen = 15 per cent.

Calculate the ratio of air to petrol consumption by weight if the volumetric analysis of the dry exhaust gas is :

CO_2 = 11.5 per cent ; CO = 1.2 per cent ; O_2 = 0.9 per cent ; N_2 = 86 per cent.

Also find percentage excess air.

Solution.

Name of gas	*Volume per m³ of flue gas* (x)	*Molecular weight* (y)	*Relative weight* $z = x \times y$	*Weight per kg of flue gas* $\frac{z}{\Sigma z}$
CO_2	0.115	44	5.06	0.1700
CO	0.012	28	0.336	0.0113
O_2	0.009	32	0.288	0.0096
N_2	0.86	28	24.08	0.8091
			$\Sigma z = 29.76$	

∴ Weight of carbon per kg of flue gas

= Weight of carbon in 0.17 kg of CO_2 + Weight of carbon in 0.0113 kg of CO

$$= \frac{3}{11} \times 0.17 + \frac{3}{7} \times 0.0113 = 0.0512 \text{ kg}$$

∴ Weight of dry flue gas per kg of fuel $= \dfrac{0.85}{0.0512} = 16.6$ kg

Vapour of combustion $= 9 \times 0.15 = 1.35$ kg

Total weight of gas $= 16.6 + 1.35 = 17.95$ kg per kg of fuel

∴ Air supplied $= (17.95 - 1) = 16.95$ kg/kg of fuel

∴ *Ratio of air to petrol* = **16.95 : 1. (Ans.)**

Stoichiometric air $= \left[\left(0.85 \times \dfrac{8}{3}\right) + (0.15 \times 8)\right] \times \dfrac{100}{23}$

$= 15.07$ kg per kg of fuel

∴ Excess air $= 16.95 - 15.07 = 1.88$ kg

∴ *Percentage excess air* $= \dfrac{1.88}{15.07} \times 100 = \mathbf{12.47\%}$. **(Ans.)**

☞**Example 11.8.** *A sample of fuel has the following percentage composition : Carbon = 86 per cent ; Hydrogen = 8 per cent ; Sulphur = 3 per cent ; Oxygen = 2 per cent ; Ash = 1 per cent.*

For an air-fuel ratio of 12 : 1, calculate :

(i) Mixture strength as a percentage rich or weak.

(ii) Volumetric analysis of the dry products of combustion.

Solution.

Element, wt. (kg)	*O_2 reqd. (kg)*
C = 0.86	$0.86 \times \frac{8}{3} = 2.29$
$H_2 = 0.08$	$0.08 \times 8 = 0.64$
S = 0.03	$0.03 \times \frac{1}{1} = 0.03$
$O_2 = 0.02$	
	Total $O_2 = 2.96$

Weight of oxygen to be supplied per kg of fuel = 2.96 – 0.02 = 294 kg.

Weight of minimum air required for complete combustion = $\frac{2.94 \times 100}{23}$ = 12.78 kg

Hence "*correct*" fuel-air ratio $= \frac{1}{12.78} : 1$

But actual ratio is $\frac{1}{12} : 1$.

(*i*) **Mixture strength** $= \frac{12.78}{12} \times 100 = 106.5\%$

This show that mixture is **6.5% rich. (Ans.)**

Deficient amount of air = 12.78 – 12 = 0.78 kg

Amount of air saved by burning 1 kg of C to CO instead of CO_2

$$= \text{Oxygen saved} \times \frac{100}{23}$$

$$= \left[\frac{8}{3}(CO_2) - \frac{4}{3}(CO)\right] \times \frac{100}{23} = 5.8 \text{ kg}$$

Hence $\frac{0.78}{5.8}$ = 0.134 kg of carbon burns to CO and as such 0.86 – 0.134 = 0.726 kg of carbon burns to CO_2.

$\therefore$ CO formed $= 0.134 \times \frac{7}{3} = 0.313$ kg

CO_2 formed $= 0.726 \times \frac{11}{3} = 2.662$ kg

N_2 supplied $= 12 \times 0.77 = 9.24$ kg

SO_2 formed $= 0.03 \times 2 = 0.06$ kg.

(*ii*) **The percentage composition of dry flue gases** is given as below :

Dry products	*Weight (kg)* x	*Molecular weight* y	*Proportional volume* $z = \frac{x}{y}$	*Percentage volume* $\frac{z}{\Sigma z} \times 100$
CO	0.313	28	0.0112	**2.78 per cent (Ans.)**
CO_2	2.662	44	0.0605	**15.03 per cent (Ans.)**
N_2	9.24	28	0.3300	**81.97 per cent (Ans.)**
SO_2	0.06	64	0.0009	**0.22 per cent (Ans.)**
			$\Sigma z = 0.4026$	

Example 11.9. *A fuel ($C_{10}H_{22}$) is burnt using an air-fuel ratio of 13 : 1 by weight. Determine the complete volumetric analysis of the products of combustion, assuming that the whole amount of hydrogen burns to form water vapour and there is neither any free oxygen nor any free carbon. The carbon burns to CO_2 and CO.*

Air contains 77% of nitrogen and 23% of oxygen by weight.

Solution. Combustion equation is :

$$2C_{10}H_{22} + 31O_2 = 20CO_2 + 22H_2O$$

$$2 \times 142 + 31 \times 32 = 20 \times 44 + 22 \times 18$$

or

$$284 + 992 = 880 + 396$$

∴ Air required for complete combustion

$$= \frac{992 \times 100}{284 \times 23} = 15.2 \text{ kg/kg of fuel}$$

Air actually supplied $= 13$ kg/kg of fuel

∴ Deficiency of air $= 15.2 - 13 = 2.2$ kg/kg of fuel

Also 1 kg of C requires $\frac{4}{3} \times \frac{100}{23} = 5.8$ kg of less air to burn to CO instead of CO_2.

Hence $\frac{2.2}{5.8} = 0.379$ kg C is burnt to CO ;

and $\left(\frac{12 \times 10}{142}\right) - 0.379 = 0.466$ kg of C is burnt to CO_2.

Weight of CO_2 formed $= 0.466 \times \frac{11}{3} = 1.708$ kg

Weight of CO formed $= 0.379 \times \frac{7}{3} = 0.884$ kg

Weight of H_2O formed $= \left(\frac{22}{142}\right) \times 9 = 1.394$ kg

Weight of N_2 from air $= 13 \times 0.77 = 10.01$ kg.

The percentage composition of products of combustion

Products of combustion	*Weight* x	*Molecular weight* y	*Proportional volume* $z = \frac{x}{y}$	*Percentage* $= \frac{z}{\Sigma z} \times 100$
CO_2	1.708	44	0.0388	**7.678 per cent (Ans.)**
CO	0.884	28	0.0316	**6.254 per cent (Ans.)**
H_2O	1.394	18	0.0774	**15.317 per cent (Ans.)**
N_2	10.01	28	0.3575	**70.750 per cent (Ans.)**
			$\Sigma z = 0.5053$	

Example 11.10. *Find an expression for the weight of air supplied per kg of fuel when carbon content of the fuel and the volumetric analysis of the fuel gas is known.*

Show that carbon burnt to CO per kg of fuel is $\frac{(CO)(C)}{CO + CO_2}$, *where CO and* CO_2 *represent percentage monoxide and carbon dioxide in dry flue gas by volume and C represents percentage by weight of carbon that is actually burnt.*

Solution. Let C = Percentage of carbon, by weight, in the fuel burnt,

CO_2 = Percentage by volume of carbon dioxide in the dry flue gas,

CO = Percentage by volume of carbon monoxide in the dry flue gas, and

N_2 = Percentage by volume of nitrogen in the dry flue gas.

Consider 100 m^3 of the gas.

The actual weight of CO_2 $= CO_2 \times a \times b \times 44$ in 100 m^3 of gas

The actual weight of CO $= CO \times a \times b \times 28$ in 100 m^3 of gas

The actual weight of N_2 $= N_2 \times a \times b \times 28$ in 100 m^3 of gas

The molecular weight of CO = 28

The molecular weight of N_2 = 28

Total mass of carbon per 100 m^3 of flue gas

$$= [a \times b \times 44 \times CO_2] \times \frac{12}{44} + [\,a \times b \times 28 \times CO] \times \frac{12}{28}$$

The reaction producing CO_2 and CO are

$$\begin{array}{ccc} C + & O_2 = & CO_2 \\ 12 & 32 & 44 \end{array}$$

$$\therefore \qquad \text{C per kg of } CO_2 = \frac{12}{44}$$

Also

$$\begin{array}{ccc} 2C + & O_2 = & 2CO \\ 24 & 32 & 56 \end{array}$$

$$\therefore \qquad \text{C per kg of CO} = \frac{12}{28}$$

All the carbon available in carbon dioxide and carbon monoxide comes from carbon present in the fuel. Also it can be safely assumed that all nitrogen comes from air (*this analysis being not applicable to the fuels containing nitrogen*).

Total carbon in the flue gas $= a \times b \times 12\ (CO_2 + CO)$ per kg of carbon in the flue gas

$$= \frac{a \times b \times 28 \times N_2}{a \times b \times 12(CO + CO_2)} \qquad ...(i)$$

For every kg of fuel supplied for combustion $\frac{C}{100}$ kg of carbon pass out of the flue, whence the weight of N_2 per kg of fuel is

$$\frac{28 N_2}{12(CO + CO_2)} \times \frac{C}{100} \text{ kg} \qquad ...(ii)$$

If all this nitrogen has come from the atmosphere, then air supplied per kg of fuel

$$= \frac{28 \times N_2 \times C \times 100}{12(CO + CO_2) \times 100 \times 77} \qquad ...(iii)$$

(since air contains 77% nitrogen by weight)

$$\therefore \textit{ Air per kg of fuel} = \frac{N_2 \times C}{\frac{100 \times 77 \times 12}{28 \times 100}(CO + CO_2)} = \frac{\mathbf{N_2 \times C}}{\mathbf{33(CO + CO_2)}}. \quad \textbf{(Ans.)}$$

It has been shown above that he total carbon of the flue gas is given by $a \times b \times 12\ (CO_2 + CO)$ and that in CO by $a \times b \times 12$ CO, whence the proportion of carbon burnt to CO is given by

$$\frac{a \times b \times 12 \times CO}{a \times b \times 12(CO + CO_2)} = \frac{CO}{CO + CO_2}$$

and if C is the proportion of carbon actually burnt per kg of fuel, then

$$\textbf{Carbon burnt to CO per kg of fuel} = \frac{(CO)(C)}{CO + CO_2}. \quad \textbf{Proved.}$$

☞ **Example 11.11.** *The following results were obtained in a trial on a boiler fitted with economiser :*

	CO_2	CO	O_2	N_2
Analysis of gas entering the economiser	*8.3*	*0*	*11.4*	*80.3*
Analysis of gas leaving the economiser	*7.9*	*0*	*11.5*	*80.6*

(i) Determine the air leakage into the economiser if the carbon content of the fuel is 80 per cent.

(ii) Determine the reduction in temperature of the gas due to air leakage if atmospheric temperature is 20°C and flue gas temperature is 410°C. Ash collected from ash pan is 15 per cent by weight of the fuel fired.

Take : c_p *for air = 1.005 kJ/kg K and* c_p *for flue gas = 1.05 kJ/kg K.*

Solution. (*i*) Air supplied $= \frac{N_2 \times C}{33(CO_2 + CO)}$

Air supplied on the basis of conditions at entry to the economiser

$$= \frac{80.3 \times 80}{33(8.3 + 0)} = 23.45 \text{ kg}$$

Air applied on the basis of conditions at exit

$$= \frac{80.6 \times 80}{33(7.9 + 0)} = 24.73 \text{ kg}$$

∴ *Air leakage* = 24.73 – 23.45 = **1.28 kg of air per kg of fuel. (Ans.)**

For each kg of fuel burnt, the ash collected is 15% *i.e.*, 0.15 kg.

∴ Weight of fuel passing up the chimney = 1 – 0.15 = 0.85 kg

∴ Total weight of products

= Weight of air supplied per kg of fuel

+ Weight of fuel passing through chimney per kg of fuel

= 23.45 + 0.85 = 24.3 kg

Heat in flue gases per kg of coal

= Weight of flue gases × Specific heat × Temperature rise above 0°C

= 24.3 × 1.05 × (410 – 0) = 10461 kJ

Heat in leakage air = Weight of leakage air × Specific heat

× Temperature rise of air above 0°C

= 1.28 × 1.005 × (20 – 0) = 25.73 kJ.

We can still consider, in the mixture, the gas and the air as separate and having their own specific heats, but sharing a common temperature t.

For heat balance :

$$(1.005 \times 1.28 + 24.3 \times 1.05)\, t = 10461 + 25.73$$

$$26.8\, t = 10486.73$$

$$\therefore \quad t = 391.3°\text{C}$$

∴ *Fall in temperature as a result of the air leakage into the economiser*

= 410 – 391.3 = **18.7°C. (Ans.)**

AIR-FUEL RATIO AND ANALYSIS OF PRODUCTS OF COMBUSTION

Example 11.12. *The chemical formula for alcohol is* C_2H_6O*. Calculate the stoichiometric air/fuel ratio by mass and the percentage composition of the products of combustion per kg of* C_2H_6O. **(GATE 1998)**

Solution. Chemical equation for complete combustion of given fuel can be written as follows :

$$C_2H_6O + 3O_2 \longrightarrow 2CO_2 + 3H_2O$$

$$(1 \times 46) + (3 \times 32) = (2 \times 44) + (3 \times 18)$$

For complete combustion of 1 kg of C_2H_6O, oxygen required

$$= \frac{3 \times 32}{46} = 2.087 \text{ kg of oxygen}$$

$$= 2.087 \times \frac{100}{23} = 9.074 \text{ kg of air}$$

A : F ratio for complete combustion = **9.074 : 1. Ans.**

Also 46 kg of fuel produces products of combustion = 88 + 54 = 142 kg

$$\left[\therefore \ 1 \text{ kg of fuel produces } \frac{142}{46} = 3.087 \text{ kg of products of combustion } (i.e., \ CO_2 \text{ and } H_2O)\right]$$

Hence **CO_2** produced by fuel = $\frac{88}{46} \times 100 = 1.913$ or **191.3%. (Ans.)**

H_2O produced by fuel = $\frac{54}{46} \times 100 = 1.174$ or **117.4%. (Ans.)**

Example 11.13. *Calculate the amount of theoretical air required for the combustion of 1 kg of acetylene (C_2H_2) to CO_2 and H_2O.*

Solution. For combustion of acetylene (C_2H_2) the stoichiometric equation is written as

$$C_2H_2 + x\,O_2 \longrightarrow a\,CO_2 + b\,H_2O \qquad ...(i)$$

Balancing the *carbon atoms* on both sides of the combustion eqn. (*i*), we get

$$2C = a\,C \quad i.e., \quad a = 2$$

Now balancing *hydrogen atoms* on both sides, we get

$$2H = 2bH$$
$$b = 1$$

Thus, eqn. (*i*) becomes

$$C_2H_2 + x\,O_2 \longrightarrow 2CO_2 + H_2O$$

Now, balancing *oxygen atoms* in the above equation

$$2x = 2 \times 2 + 1 = 5 \quad i.e., \quad x = 2.5$$

Hence, the final combustion eqn. (*i*) is

$$C_2H_2 + 2.5O_2 \longrightarrow 2CO_2 + H_2O \qquad ...(ii)$$

Thus, for combustion of C_2H_2 in air, we get

$$C_2H_2 + 2.5O_2 + 2.5\left(\frac{79}{21}\right)N_2 \longrightarrow 2CO_2 + H_2O + 2.5\left(\frac{79}{21}\right)N_2$$

On a mass basis, this becomes

$$(2 \times 12 + 2 \times 1)\,C_2H_2 + 2.5\,(2 \times 16)\,O_2 + 2.5\left(\frac{79}{21}\right)(2 \times 14)\,N_2$$

$$\longrightarrow 2\,(12 + 2 \times 16)\,CO_2 + (1 \times 2 + 1 \times 16)\,H_2O + 2.5\left(\frac{79}{21}\right)(2 \times 14)\,N_2$$

$$26 \text{ kg } C_2H_2 + 80 \text{ kg } O_2 + 263.3\ N_2 \longrightarrow 88 \text{ kg } CO_2 + 18 \text{ kg } H_2O + 263.3 \text{ kg } N_2$$

or $$1 \text{ kg } C_2H_2 + 3.076 \text{ kg } O_2 + 10.12 \text{ kg } N_2 \longrightarrow 3.38 \text{ kg } CO_2 + 0.69 \text{ kg } H_2O + 10.12 \text{ kg } N_2$$

i.e., Amount of air = 3.076 + 10.12 = 13.196 kg of air per kg of C_2H_2

Hence amount of theoretical air required for combustion of 1 kg acetylene = 13.196 kg. (Ans.)

Example 11.14. *Determine the gravimetric analysis of the products of complete combustion of acetylene with 200 per cent stoichiometric air.*

Solution. The stoichiometric air equation (Example 11.3) is written as :

$$C_2H_2 + 2.5\,O_2 + 2.5\left(\frac{79}{21}\right)N_2 \longrightarrow 2CO_2 + H_2O + 2.5\left(\frac{79}{21}\right)N_2$$

If *200 per cent* stoichiometric air is used, the combustion equation for C_2H_2 becomes

$$C_2H_2 + (2)(2.5)\,O_2 + (2)(2.5)\left(\frac{79}{21}\right)N_2 \longrightarrow 2CO_2 + H_2O + (2)(2.5)\left(\frac{79}{21}\right)N_2 + (1)(2.5)\,O_2$$

$$(2 \times 12 + 2 \times 1) \text{ kg } C_2H_2 + (2)(2.5)(2 \times 16) \text{ kg } O_2 + (2)(2.5)\left(\frac{79}{21}\right)(2 \times 14) \text{ kg } N_2$$

$$\longrightarrow 2\,(12 + 2 \times 16)\text{ kg } CO_2 + (2 \times 1 + 16)\text{ kg } H_2O + (2)\,(2.5)\left(\frac{79}{21}\right)(2 \times 14)\text{ kg } N_2$$

$$+ (1)\,(2.5)\,(2 \times 16)\text{ kg } O_2$$

or $$26\text{ kg } C_2H_2 + 160\text{ kg } O_2 + 526.6\text{ kg } N_2 \longrightarrow 88\text{ kg } CO_2 + 18\text{ kg } H_2O + 526.6\text{ kg } N_2 + 80\text{ kg } O_2$$

or $$1\text{ kg } C_2H_2 + 6.15\text{ kg } O_2 + 20.25\text{ kg } N_2 \longrightarrow 3.38\text{ kg } CO_2 + 0.69\text{ kg } H_2O + 20.25\text{ kg } N_2 + 3.07\text{ kg } O_2$$

Thus for 1 kg of fuel, the products will consist of

CO_2 = 3.38 kg
H_2O = 0.69 kg
O_2 = 3.07 kg
N_2 = 20.25 kg

Total mass of products = 27.39 kg

∴ Mass fractions are :

$$CO_2 = \frac{3.38}{27.39} = 0.123$$

$$H_2O = \frac{0.69}{27.39} = 0.025$$

$$O_2 = \frac{3.07}{27.39} = 0.112$$

$$N_2 = \frac{20.25}{27.39} = 0.739.$$

Hence the gravimetric analysis of the complete combustion is :

CO_2 = 12.3%, H_2O = 2.5%, O_2 = 11.2%, N_2 = 73.9%. (Ans.)

Example 11.15. *Calculate the theoretical air-fuel ratio for the combustion of octane, C_8H_{18}. The combustion equation is :*

$$C_8H_{18} + 12.5\,O_2 + 12.5\left(\frac{79}{21}\right)N_2 \longrightarrow 8CO_2 + 9H_2O + 12.5\left(\frac{79}{21}\right)N_2.$$

Solution. The air-fuel ratio on a mole basis is

$$A/F = \frac{12.5 + 12.5\left(\frac{79}{21}\right)}{1} = 59.5 \text{ mol air/mol fuel}$$

The theoretical air-fuel ratio on a mass basis is found by introducing the molecular weight of the air and fuel

$$A/F = \frac{59.5(28.97)}{(8 \times 12 + 1 \times 18)} = \textbf{15.08 kg air/kg fuel.}\ \textbf{(Ans.)}$$

Example 11.16. *One kg of octane (C_8H_{18}) is burned with 200% theoretical air. Assuming complete combustion determine :*

(i) Air-fuel ratio

(ii) Dew point of the products at a total pressure 100 kPa.

Solution. The equation for the combustion of C_8H_{18} with theoretical air is

$$C_8H_{18} + 12.5\,O_2 + 12.5\left(\frac{79}{21}\right)N_2 \longrightarrow 8CO_2 + 9H_2O + 12.5\left(\frac{79}{21}\right)N_2$$

For *200% theoretical air* the combustion equation would be

$$C_8H_{18} + (2)(12.5)\,O_2 + (2)(12.5)\left(\frac{79}{21}\right) N_2$$

$$\longrightarrow 8CO_2 + 9H_2O + (1)(12.5)\,O_2 + (2)(12.5)\left(\frac{79}{21}\right) N_2$$

Mass of fuel = (1) (8 × 12 + 1 × 18) = 114 kg/mole

Mass of air = $(2)(12.5)\left(1+\frac{79}{21}\right) 28.97$ = 3448.8 kg/mole of fuel

(*i*) **Air-fuel ratio :**

Air-fuel ratio, $A/F = \frac{\text{Mass of air}}{\text{Mass of fuel}} = \frac{3448.8}{114} = 30.25$

i.e., **A/F = 30.25. (Ans.)**

(*ii*) **Dew point of the products, t_{dp} :**

Total number of moles of products

$$= 8 + 9 + 12.5 + (2)(12.5)\left(\frac{79}{21}\right) = 123.5 \text{ moles/mole fuel}$$

Mole fraction of $H_2O = \frac{9}{123.5} = 0.0728$

Partial pressure of $H_2O = 100 \times 0.0728 = 7.28$ kPa

The saturation temperature corresponding to this pressure is 39.7°C which is also the dew-point temperature.

Hence t_{dp} **= 39.7°C. (Ans.)**

Note. The water condensed from the products of combustion usually contains some dissolved gases and therefore may be quite *corrosive*. For this reason the products of combustion are often *kept above the dew point until discharged to the atmosphere.*

Example 11.17. *One kg of ethane (C_2H_6) is burned with 90% of theoretical air. Assuming complete combustion of hydrogen in the fuel determine the volumetric analysis of the dry products of combustion.*

Solution. The complete combustion equation for C_2H_6 is written as :

$$C_2H_6 + 3.5O_2 \longrightarrow 2CO_2 + 3H_2O$$

The combustion equation for C_2H_6 for 90% theoretical air is written as :

$$C_2H_6 + (0.9)(3.5)\,O_2 + (0.9)(3.5)\left(\frac{79}{21}\right) N_2 \longrightarrow a\,CO_2 + b\,CO + 3H_2O + (0.9)(3.5)\left(\frac{79}{21}\right) N_2$$

By balancing *carbon atoms* on both the sides, we get

$$2 = a + b \qquad ...(i)$$

By balancing *oxygen atoms* on both the sides, we get

$$(0.9)(3.5)(2) = 2a + b + 3 \qquad ...(ii)$$

Substituting the value of b (= 2 − a) from eqn. (*i*) in eqn. (*ii*), we get

$$(0.9)(3.5)(2) = 2a + 2 - a + 3$$

$$6.3 = a + 5$$

$$a = 1.3$$

and $$b = 2 - a = 2 - 1.3 = 0.7$$

Thus the combustion equation becomes :

$$C_2H_6 + (0.9)(3.5)\,O_2 + (0.9)(3.5)\left(\frac{79}{21}\right)N_2$$
$$\longrightarrow 1.3CO_2 + 0.7CO + 3H_2O + (0.9)(3.5)\left(\frac{79}{21}\right)N_2$$

Total number of moles of *dry products* of combustion

$$= 1.3 + 0.7 + (0.9)(3.5)\left(\frac{79}{21}\right)$$
$$= 1.3 + 0.7 + 11.85 = 13.85 \text{ moles/mole of fuel}$$

Volumetric analysis of dry products of combustion is as follows :

$$CO_2 = \frac{1.3}{13.85} \times 100 = 9.38\%. \quad \textbf{(Ans.)}$$

$$CO = \frac{0.7}{13.85} \times 100 = 5.05\%. \quad \textbf{(Ans.)}$$

$$N_2 = \frac{11.85}{13.85} \times 100 = 85.56\%. \quad \textbf{(Ans.)}$$

☞ **Example 11.18.** *Methane (CH_4) is burned with atmospheric air. The analysis of the products on a 'dry' basis is as follows :*

$$CO_2 = 10.00\%,\ O_2 = 2.37\%,\ CO = 0.53\%,\ N_2 = 87.10\%.$$

(i) Determine the combustion equation ; (ii) Calculate the air-fuel ratio ;

(iii) Percent theoretical air.

Solution. (*i*) **Combustion equation :**

From the analysis of the products, the following equation can be written, keeping in mind that this analysis is on a *dry basis.*

$$x\,CH_4 + y\,O_2 + z\,N_2 \longrightarrow 10.0\,CO_2 + 0.53\,CO + 2.37\,O_2 + a\,H_2O + 87.1\,N_2$$

To determine all the unknown co-efficients let us find balance for each of the elements.

Nitrogen balance : $z = 87.1$

Since all the nitrogen comes from the air,

$$\frac{z}{y} = \frac{79}{21}\ ;\ y = \frac{87.1}{(79/21)} = 23.16$$

Carbon balance : $x = 10.00 + 0.53 = 10.53$

Hydrogen balance : $a = 2x = 2 \times 10.53 = 21.06$

Oxygen balance. All the unknown co-efficients have been solved for, and in this case the oxygen balance provides a check on the accuracy. Thus, y can also be determined by an oxygen balance

$$y = 10.00 + \frac{0.53}{2} + 2.37 + \frac{21.06}{2} = 23.16$$

Substituting these values for x, y, z and a, we have,

$$10.53\,CH_4 + 23.16\,O_2 + 87.1\,N_2 \longrightarrow 10.0\,CO_2 + 0.53\,CO + 2.37\,O_2 + 21.06\,H_2O + 87.1\,N_2$$

Dividing both sides by 10.53, we get the **combustion equation** per mole of fuel,

$$\mathbf{CH_4 + 2.2\,O_2 + 8.27\,N_2 \longrightarrow 0.95\,CO_2 + 0.05\,CO + 2H_2O + 0.225\,O_2 + 8.27\,N_2.} \quad \textbf{(Ans.)}$$

(*ii*) **Air-fuel ratio A/F :**

The air-fuel ratio on a *mole basis* is

$$2.2 + 8.27 = \mathbf{10.47\ moles\ air/mole\ fuel.}\quad \textbf{(Ans.)}$$

The air-fuel ratio on a *mass basis* is found by introducing the molecular weights

$$A/F = \frac{10.47 \times 28.97}{(12 + 1 \times 4)} = \mathbf{18.96\ kg\ air/kg\ fuel.}\quad \textbf{(Ans.)}$$

The theoretical air-fuel ratio is found by writing the combustion equation for theoretical air,

$$CH_4 + 2O_2 + 2\left(\frac{79}{21}\right)N_2 \longrightarrow CO_2 + 2H_2O + (2)\left(\frac{79}{21}\right)N_2$$

$$A/F_{theor.} = \frac{\left[2 + (2)\left(\frac{79}{21}\right)\right]28.97}{(12 + 1 \times 4)} = \mathbf{17.24\ kg\ air/kg\ fuel.}\quad \textbf{(Ans.)}$$

(*iii*) **Percent theoretical air :**

$$\text{Per cent theoretical air} = \frac{18.96}{17.24} \times 100 = \mathbf{110\%.}\quad \textbf{(Ans.)}$$

Example 11.19. *The gravimetric analysis of a sample of coal is given as 82% C, 10% H_2 and 8% ash. Calculate :*

(*i*) *The stoichiometric A/F ratio ;* (*ii*) *The analysis of the products by volume ;*

Solution. (*i*) **The stoichiometric A/F ratio :**

1 kg of coal contains 0.82 kg C and 0.10 kg H_2.

$$\therefore\ \text{1 kg of coal contains } \frac{0.82}{12} \text{ moles C and } \frac{0.10}{2} \text{ moles } H_2$$

Let the oxygen required for complete combustion = x moles

$$\text{Then the nitrogen supplied with the oxygen} = x \times \frac{79}{21} = 3.76x \text{ moles}$$

For 1 kg of coal the combustion equation is therefore as follows :

$$\frac{0.82}{12}C + \frac{0.10}{2}H_2 + x\ CO_2 + 3.76x\ N_2 \longrightarrow a\ CO_2 + b\ H_2O + 3.76\ x\ N_2$$

Then, **Carbon balance** : $\frac{0.82}{12} = a$ $\quad\therefore\ a = 0.068$ moles

Hydrogen balance : $2 \times \frac{0.10}{2} = 2b$ $\quad\therefore\ b = 0.05$ moles

Oxygen balance : $2x = 2a + b$ $\quad\therefore\ x = \left(\frac{2 \times 0.068 + 0.05}{2}\right) = 0.093$ moles

The mass of 1 mole of oxygen is 32 kg, therefore, the mass of O_2 supplied per kg of coal = 32 × 0.093 = 2.976 kg

i.e., $\quad$ *Stoichiometric A/F ratio* $= \frac{2.976}{0.233} = \mathbf{12.77}$ **(Ans.)**

(where air is assumed to contain 23.3% O_2 and 76.7% N_2 by mass)

Total moles of products $= a + b + 3.76x = 0.068 + 0.05 + 3.76 \times 0.093 = 0.467$ moles

Hence *the analysis of the products* is

$$\mathbf{CO_2} = \frac{0.068}{0.467} \times 100 = \mathbf{14.56\%.} \quad \mathbf{(Ans.)}$$

$$\mathbf{H_2} = \frac{0.05}{0.467} \times 100 = \mathbf{10.7\%.} \quad \mathbf{(Ans.)}$$

$$\mathbf{N_2} = \frac{(3.76 \times 0.093)}{0.467} \times 100 = \mathbf{74.88\%.} \quad \mathbf{(Ans.)}$$

☞**Example 11.20.** *Calculate the stoichiometric air-fuel ratio for the combustion of a sample of dry anthracite of the following composition by mass :*

Carbon (C) = 88 per cent *Hydrogen (H_2) = 4 per cent*
Oxygen (O_2) = 3.5 per cent *Nitrogen (N_2) = 1 per cent*
Sulphur (S) = 0.5 per cent *Ash = 3 per cent*

If 30 per cent excess air is supplied determine :

(i) Air-fuel ratio ;

(ii) Wet dry analysis of the products of combustion by volume.

Solution. Stoichiometric air-fuel (A/F) ratio :

In case of a fuel with several constituents a *tubular method* is advisable, as shown below. Each constituent is taken separately and the amount of oxygen required for complete combustion is found from the chemical equation. The oxygen in the fuel is included in the column headed *'oxygen required'* as a *negative quantity.*

	Mass per kg coal	*Combustion equation*	*Oxygen required per kg of coal*	*Products per kg of coal*
C	0.88	$C + O_2 \longrightarrow CO_2$ 12 kg + 32 kg ⟶ 44 kg	$0.88 \times \frac{32}{12} = 2.346$ kg	$0.88 \times \frac{44}{12} = 3.23$ kg CO_2
H_2	0.04	$2H_2 + O_2 \longrightarrow 2H_2O$ 1 kg + 8 kg ⟶ 9 kg	$0.04 \times 8 = 0.32$ kg	$0.04 \times 9 = 0.36$ kg H_2O
O_2	0.035	—	-0.035 kg	—
N_2	0.01	—	—	0.01 kg N_2
S	0.005	$S + O_2 \longrightarrow SO_2$ 32 kg + 32 kg ⟶ 64 kg	$0.005 \times \frac{32}{32} = 0.005$ kg	$0.005 \times \frac{64}{32} = 0.01$ kg SO_2
Ash	0.03	—	—	—
			Total $O_2 = 2.636$ kg	

From table :

O_2 required per kg of coal = 2.636 kg

$\therefore$ Air required per kg of coal $= \frac{2.636}{0.233} = 11.31$ kg

(where air is assumed to contain 23.3% O_2 *by mass*)

N_2 associated with this air = 0.767 × 11.31 = 8.67 kg

$\therefore$ Total N_2 in products = 8.67 + 0.01 = 8.68 kg

The stoichiometric A/F ratio = 11.31/1. (Ans.)

When 30 per cent excess air is used :

(*i*) **Actual A/F ratio :**

$$\textbf{Actual A/F ratio} = 11.31 + 11.31 \times \frac{30}{100} = \textbf{14.7/1. Ans.}$$

(*ii*) **Wet and dry analyses of products of combustion by volume :**

As per actual A/F ratio, N_2 supplied = 0.767 × 14.7 = 11.27 kg

Also O_2 supplied = 0.233 × 14.7 = 3.42 kg

(where air is assumed to contain N_2 = 76.7% and O_2 = 23.3)

In the products then, we have

$$N_2 = 11.27 + 0.01 = 11.28 \text{ kg}$$

and

$$\text{excess } O_2 = 3.42 - 2.636 = 0.78 \text{ kg}$$

The products are entered in the following table and the analysis by volume is obtained :

- In column 3 the percentage by mass is given by the mass of each product divided by the total mass of 15.66 kg.
- In column 5 the moles per kg of coal are given by equation $n = \frac{m}{M}$. The total of column 5 gives the total moles of wet products per kg of coal, and by subtracting the moles of H_2O from this total, the total moles of dry products is obtained as 0.5008.
- Column 6 gives the proportion of each constituent of column 5 expressed as a percentage of the total moles of the wet products.
- Similarly column 7 gives the percentage by volume of the dry products.

Product	*Mass / kg coal*	*% by mass*	*M*	*Moles / kg coal*	*% by vol. wet*	*% by vol. dry*
1	2	3	4	5	6	7
CO_2	3.23	20.62	44	0.0734	14.10	14.66
H_2O	0.36	2.29	18	0.0200	3.84	—
SO_2	0.01	0.06	64	0.0002 (say)	0.04	0.04
O_2	0.78	4.98	32	0.0244	4.68	4.87
N_2	11.28	72.03	28	0.4028	77.34	80.43
	15.66 kg			Total wet = 0.5208 $- H_2O$ = 0.0200 Total dry = 0.5008	100.00	100.00 **(Ans.)**

Example 11.21. *The following analysis relate to coal gas :*

H_2 = *50.4 per cent* CO = *17 per cent*

CH_4 = *20 per cent* C_4H_8 = *2 per cent*

O_2 = *0.4 per cent* N_2 = *6.2 per cent*

CO_2 = *4 per cent.*

(*i*) *Calculate the stoichiometric A/F ratio.*

(*ii*) *Find also the wet and dry analyses of the products of combustion if the actual mixture is 30 per cent weak.*

Solution. The example is solved by a tabular method ; a specimen calculation is given below :

For CH_4 :

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

i.e., 1 mole CH_4 + 2 moles $O_2 \longrightarrow$ 1 mole CO_2 + 2 moles H_2O

There are 0.2 moles of CH_4 per mole of the coal gas, hence

0.2 moles CH_4 + 0.2 × 2 moles $O_2 \longrightarrow$ 0.2 moles CO_2 + 0.2 × 2 moles H_2O

∴ O_2 required for the CH_4 in the coal gas = 0.4 moles per mole of coal gas.

The oxygen in the fuel (0.004 moles) is included in column 4 as a *negative quantity.*

Product	*Moles/mole fuel*	*Combustion equation*	*O_2 moles / mole fuel*	*Products CO_2*	*H_2O*
1	*2*	*3*	*4*	*5*	*6*
H_2O	0.504	$2H_2 + O_2 \longrightarrow 2H_2O$	0.252	—	0.504
CO	0.17	$2CO + O_2 \longrightarrow 2CO_2$	0.085	0.17	—
CH_4	0.20	$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$	0.400	0.20	0.40
C_4H_8	0.02	$C_4H_8 + 6O_2 \longrightarrow 4CO_2 + 4H_2O$	0.120	0.08	0.08
O_2	0.004	—	−0.004	—	—
N_2	0.062	—	—	—	—
CO_2	0.04	—	—	0.04	—
			Total = 0.853	0.49	0.984

(*i*) **Stoichiometric A/F ratio :**

$$\text{Air required} = \frac{0.853}{0.21} = 4.06 \text{ moles/mole of fuel}$$

(where air is assumed to contain 21% O_2 by volume)

∴ **Stoichiometric A/F ratio = 4.06/1 by volume. (Ans.)**

(*ii*) **Wet and dry analyses of the products of combustion if the actual mixture is 30% weak :**

Actual A/F ratio with 30% weak mixture

$$= 4.06 + \frac{30}{100} \times 4.06 = 1.3 \times 4.06 = 5.278/1$$

Associated N_2 = 0.79 × 5.278 = 4.17 moles/mole fuel

Excess oxygen = 0.21 × 5.278 − 0.853 = 0.255 moles

Total moles of N_2 in products = 4.17 + 0.062 = 4.232 moles/mole fuel.

Analysis by volume of wet and dry products :

Product	*Moles/mole fuel*	*% by vol. (dry)*	*% by vol. (wet)*
CO_2	0.490	9.97	8.31
H_2O	0.984	—	16.68
O_2	0.255	5.19	4.32
N_2	4.170	84.84	70.69
	Total wet = 5.899	100.00	100.00
	$-H_2O$ = 0.984		**(Ans.)**
	Total dry = 4.915		

Example 11.22. *Find the stoichiometric air-fuel ratio for the combustion of ethyl alcohol (C_2H_6O), in a petrol engine. Calculate the air-fuel ratios for the extreme mixture strengths of 80 per cent and 130 per cent. Determine also the wet and dry analyses by volume of the exhaust gas for each mixture strength.*

Solution. The equation for combustion of ethyl alcohol is as follows :

$$C_2H_6O + 3O_2 \longrightarrow 2CO_2 + 3H_2O$$

— Since there are two atoms of carbon in each mole of C_2H_6O then there must be two moles of CO_2 in the products, giving two atoms of carbon on each side of the equation.

— Similarly, since there are six atoms of hydrogen in each mole of ethyl alcohol then there must be three moles of H_2O in the products, giving six atoms of hydrogen on each side of the equation.

— Then balancing the atoms of oxygen, it is seen that there are (2 × 2 + 3) = 7 atoms on the right hand side of the equation, hence seven atoms must appear on the left hand side of the equation. There is one atom of oxygen in ethyl alcohol, therefore a further six atoms of oxygen must be supplied, and hence three moles of oxygen are required as shown.

Since the O_2 is supplied as air, the associated N_2 must appear in the equation,

i.e., $$C_2H_6O + 3O_2 + 3 \times \frac{79}{21} N_2 \longrightarrow 2CO_2 + 3H_2O + 3 \times \frac{79}{21} N_2$$

One mole of fuel has a mass of (2 × 12 + 1 × 6 + 16) = 46 kg. Three moles of oxygen have a mass of (3 × 32) = 96 kg.

$$\therefore \quad O_2 \text{ required per kg of fuel} = \frac{96}{46} = 2.09 \text{ kg}$$

$$\therefore \quad \textbf{Stoichiometric A/F ratio} = \frac{2.09}{0.233} = \textbf{8.96/1.} \quad \textbf{(Ans.)}$$

Considering a mixture strength of 80% :

$$\text{Now, mixture strength} = \frac{\text{Stoichiometric A/F ratio}}{\text{Actual A/F ratio}}$$

i.e., $$0.8 = \frac{8.96/1}{\text{Actual A/F ratio}}$$

$$\therefore \quad \textbf{Actual A/F ratio} = \frac{8.96}{0.8} = \textbf{11.2/1.} \quad \textbf{(Ans.)}$$

This means that 1/0.8 or 1.25 times as much air is supplied as is necessary for complete combustion. The exhaust will therefore contain 0.25 stoichiometric oxygen.

i.e., $$C_2H_6O + 1.25 \left(3O_2 + 3 \times \frac{79}{21} N_2\right) \longrightarrow 2CO_2 + 3H_2O + 0.25 \times 3O_2 + 1.25 \times 3 \times \frac{79}{21} N_2$$

i.e., The products are :

$$2 \text{ moles } CO_2 + 3 \text{ moles } H_2O + 0.75 \text{ moles } O_2 + 14.1 \text{ moles } N_2$$

The total moles = 2 + 3 + 0.75 + 14.1 = 19.85

Hence wet analysis is :

$$\mathbf{CO_2} = \frac{2}{19.85} \times 100 = \textbf{10.08\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{H_2O} = \frac{3}{19.85} \times 100 = \textbf{15.11\%.} \quad \textbf{(Ans.)}$$

$$O_2 = \frac{0.75}{19.85} \times 100 = \mathbf{3.78\%}. \quad \textbf{(Ans.)}$$

$$N_2 = \frac{14.1}{19.85} \times 100 = \mathbf{71.03\%}. \quad \textbf{(Ans.)}$$

The total dry moles $= 2 + 0.75 + 14.1 = 16.85$

Hence dry analysis is :

$$CO_2 = \frac{2}{16.85} \times 100 = \mathbf{11.87\%}. \quad \textbf{(Ans.)}$$

$$O_2 = \frac{0.75}{16.85} \times 100 = \mathbf{4.45\%}. \quad \textbf{(Ans.)}$$

$$N_2 = \frac{14.1}{16.85} \times 100 = \mathbf{83.68\%}. \quad \textbf{(Ans.)}$$

Considering a mixture strength of 130% :

Now, $$1.3 = \frac{\text{Stoichiometric ratio}}{\text{Actual A/F ratio}}$$

∴ **Actual A/F ratio** $= \dfrac{8.96}{1.3} = \mathbf{6.89/1}.$ **(Ans.)**

This means that $\frac{1}{1.3}$ or 0.769 of the stoichiometric air is supplied. *The combustion cannot be complete, as the necessary oxygen is not available. It is usual to assume that all hydrogen is burned to H_2O, since hydrogen atoms have a greater affinity for oxygen than carbon atoms. The carbon in the fuel will burn to CO and CO_2, but the relative proportions have to be determined.*

Let, a = Number of moles of CO_2 in the products, and

b = Number of moles of CO in the products.

Then the combustion equation is as follows :

$$C_2H_6O + 0.769\left(3O_2 + 3 \times \frac{79}{21} N_2\right) \longrightarrow a\, CO_2 + b\, CO + 3H_2O + 0.769 \times 3 \times \frac{79}{21} N_2$$

To find a and b a balance of carbon and oxygen atoms can be made,

i.e., ***Carbon balance :***

$$2 = a + b \qquad ...(i)$$

and ***Oxygen balance :***

$$1 + 2 \times 0.769 \times 3 = 2a + b + 3$$

or $$2.614 = 2a + b \qquad ...(ii)$$

From eqns. (*i*) and (*ii*), we get

$$a = 0.614,\ b = 1.386$$

i.e., The products are : 0.614 moles CO_2 + 1.386 moles CO + 3 moles H_2O + 8.678 moles N_2

The total moles = 0.614 + 1.386 + 3 + 8.678 = 13.678.

Hence wet analysis is :

$$CO_2 = \frac{0.614}{13.678} \times 100 = \mathbf{4.49\%}. \quad \textbf{(Ans.)}$$

$$CO = \frac{1.386}{13.678} \times 100 = \mathbf{10.13\%}. \quad \textbf{(Ans.)}$$

$$H_2O = \frac{3}{13.678} \times 100 = \mathbf{21.93\%.}\ \ \textbf{(Ans.)}$$

$$N_2 = \frac{8.678}{13.678} \times 100 = \mathbf{63.45\%.}\ \ \textbf{(Ans.)}$$

The total dry moles = 0.614 + 1.386 + 8.678 = 10.678

Hence dry analysis is :

$$CO_2 = \frac{0.614}{10.678} \times 100 = \mathbf{5.75\%.}\ \ \textbf{(Ans.)}$$

$$CO = \frac{1.386}{10.678} \times 100 = \mathbf{12.98\%.}\ \ \textbf{(Ans.)}$$

$$N_2 = \frac{8.678}{10.678} \times 100 = \mathbf{81.27\%.}\ \ \textbf{(Ans.)}$$

Example 11.23. *For the stoichiometric mixture of example 11.22 calculate :*

(i) The volume of the mixture per kg of fuel at a temperature of 50°C and a pressure of 1.013 bar.

(ii) The volume of the products of combustion per kg of fuel after cooling to a temperature of 130°C at a pressure of 1 bar.

Solution. As before,

$$C_2H_6O + 3O_2 + 3 \times \frac{79}{21} N_2 \longrightarrow 2CO_2 + 3H_2O + 3 \times \frac{79}{21} N_2$$

$\therefore$ Total moles reactants $= 1 + 3 + 3 \times \frac{79}{21} = 15.3$

From equation, $pV = nR_0T$

$$V = \frac{nR_0T}{p} = \frac{15.3 \times 8.314 \times 10^3 \times (50 + 273)}{1.013 \times 10^5} = 405.6 \text{ m}^3\text{/mole of fuel}$$

In 1 mole of fuel there are (2 × 12 + 6 + 16) = 46 kg

(*i*) $\therefore$ **Volume of reactants per kg of fuel** $= \frac{405.6}{46} =$ **8.817 m^3. (Ans.)**

When the products are cooled to 130°C the H_2O exists as steam, since the temperature is well above the saturation temperature corresponding to the partial pressure of the H_2O. This must be so since the saturation temperature corresponding to the *total* pressure is 99.6°C, and the *saturation temperature decreases with pressure*. The total moles of the products is

$$= \left(2 + 3 + 3 \times \frac{79}{21}\right) = 16.3$$

From equation, $pV = nR_0T$

$$V = \frac{nR_0T}{p} = \frac{16.3 \times 8.314 \times 10^3 \times (130 + 273)}{1 \times 10^5} = 546.14 \text{ m}^3\text{/mole of fuel.}$$

(*ii*) $\therefore$ **Volume of products per kg of fuel** $= \frac{546.14}{46} =$ **11.87 m^3. (Ans.)**

☞**Example 11.24.** *The following is the composition of coal gas supplied to a gas engine :*

H_2 = 50.6 per cent ; CO = 10 per cent ; CH_4 = 26 per cent ; C_4H_8 = 4 per cent ; O_2 = 0.4 per cent ; CO_2 = 3 per cent ; N_2 = 6 per cent.